CSE/MATH 455 Supplementary Notes

Two disasters caused by computer arithmetic errors

1. The Patriot Missile Failure

On February 25, 1991, during the Gulf War, an American Patriot Missile battery in Dharan, Saudi Arabia, failed to intercept an incoming Iraqi Scud missile. The Scud struck an American Army barracks and killed 28 soliders. A report of the General Accounting office, GAO/IMTEC-92-26, entitled *Patriot Missile Defense: Software Problem Led to System Failure at Dhahran, Saudi Arabia* reported on the cause of the failure. It turns out that the cause was an inaccurate calculation of the time since boot due to computer arithmetic errors. Specifically, the time in tenths of second as measured by the system's internal clock was multiplied by 1/10 to produce the time in seconds. This calculation was performed using a 24 bit fixed point register. In particular, the value 1/10, which has a non-terminating binary expansion, was chopped at 24 bits after the radix point. The small chopping error, when multiplied by the large number giving the time in tenths of a second, lead to a significant error. Indeed, the Patriot battery had been up around 100 hours, and an easy calculation shows that the resulting time error due to the magnified chopping error was about 0.34 seconds. (The number 1/10 equals $1/2^4+1/2^5+1/2^8+1/2^9+1/2^{12}+1/2^{13}+....$ In other words, the binary expansion of 1/10 is 0.0001100110011001100110011.... Now the 24 bit register in the Patriot stored instead 0.00011001100110011001100 introducing an error of 0.0000000000000000000000011001100... binary, or about 0.000000095 decimal. Multiplying by the number of tenths of a second in 100 hours gives $0.000000095\times100\times60\times60\times10=0.34$.) A Scud travels at about 1,676 meters per second, and so travels more than half a kilometer in this time. This was far enough that the incoming Scud was outside the "range gate" that the Patriot tracked. Ironically, the fact that the bad time calculation had been improved in some parts of the code, but not all, contributed to the problem, since it meant that the inaccuracies did not cancel.

The following paragraph is excerpted from the GAO report.

The range gate's prediction of where the Scud will next appear is a function of the Scud's known velocity and the time of the last radar detection. Velocity is a real number that can be expressed as a whole number and a decimal (e.g., 3750.2563...miles per hour). Time is kept continuously by the system's

internal clock in tenths of seconds but is expressed as an integer or whole number (e.g., 32, 33, 34...). The longer the system has been running, the larger the number representing time. To predict where the Scud will next appear, both time and velocity must be expressed as real numbers. Because of the way the Patriot computer performs its calculations and the fact that its registers are only 24 bits long, the conversion of time from an integer to a real number cannot be any more precise than 24 bits. This conversion results in a loss of precision causing a less accurate time calculation. The effect of this inaccuracy on the range gate's calculation is directly proportional to the target's velocity and the length of the the system has been running. Consequently, performing the conversion after the Patriot has been running continuously for extended periods causes the range gate to shift away from the center of the target, making it less likely that the target, in this case a Scud, will be successfully intercepted.

2. The Explosion of the Ariane 5

On June 4, 1996 an unmanned Ariane 5 rocket launched by the European Space Agency exploded just forty seconds after lift-off. The rocket was on its first voyage, after a decade of development costing $7 billion. The destroyed rocket and its cargo were valued at $500 million. A board of inquiry investigated the causes of the explosion and in two weeks issued a report. It turned out that the cause of the failure was a software error in the inertial reference system. Specifically a 64 bit floating point number relating to the horizontal velocity of the rocket with respect to the platform was converted to a 16 bit signed integer. The number was larger than 32,768, the largest integer storeable in a 16 bit signed integer, and thus the conversion failed.

The report of the Inquiry Board is available [in English](#) or [in the original French](#). The following paragraphs are extracted from that report. An [interesting article](#) on the accident and its implications by James Gleick appeared in The New York Times Magazine of 1 December 1996.

On 4 June 1996, the maiden flight of the Ariane 5 launcher ended in a failure. Only about 40 seconds after initiation of the flight sequence, at an altitude of about 3700 m, the launcher veered off its flight path, broke up and exploded.

The failure of the Ariane 501 was caused by the complete loss of guidance and attitude information 37 seconds after start of the main engine ignition sequence (30 seconds after lift-off). This loss of information was due to specification and design errors in the software of the inertial reference system.

The internal SRI* software exception was caused during execution of a data conversion from 64-bit floating point to 16-bit signed integer value. The floating point number which was converted had a value greater than what could be represented by a 16-bit signed integer.

*SRI stands for Système de Référence Inertielle or Inertial Reference System.

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