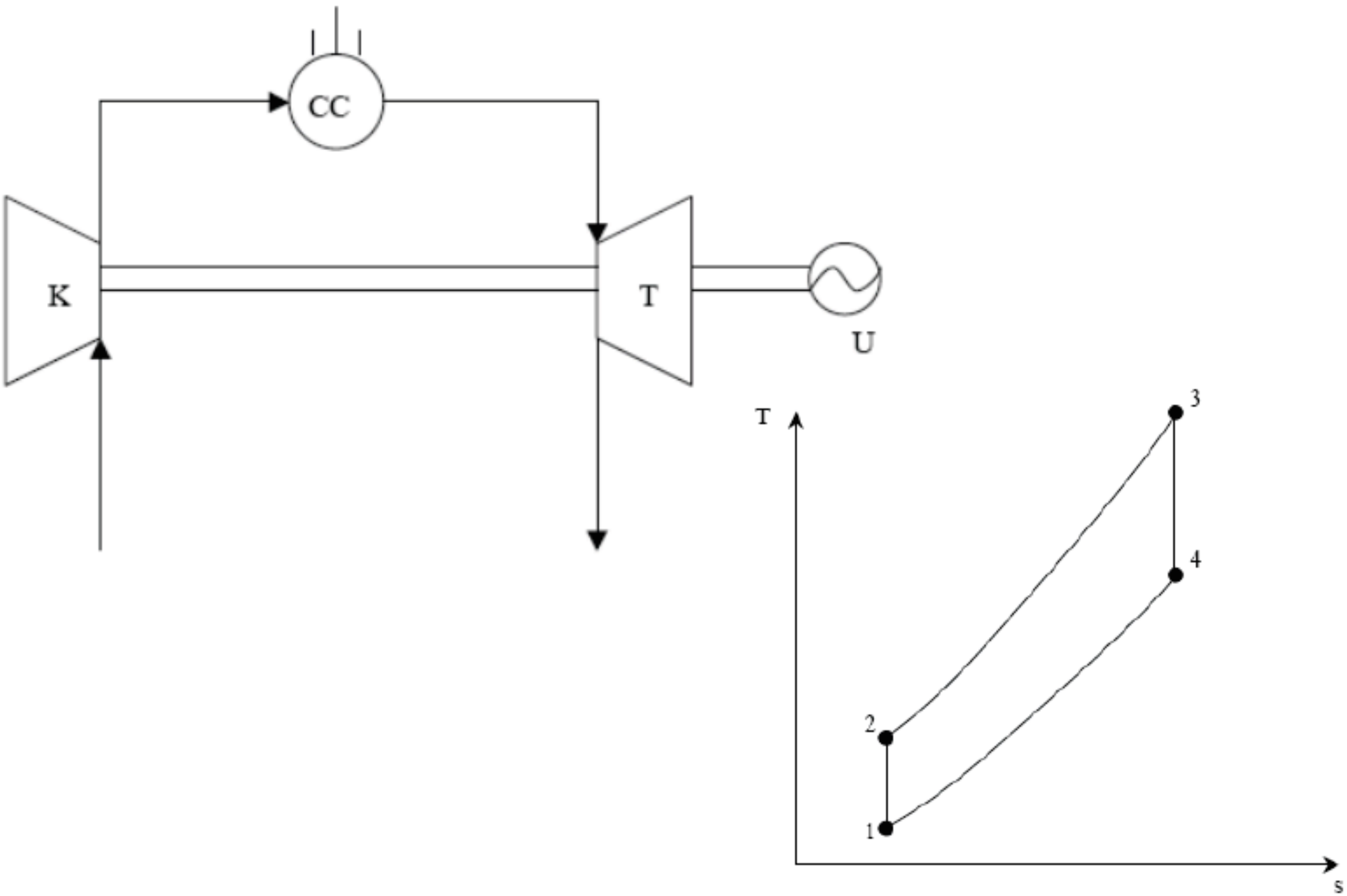
Thermodynamics of gas turbines

Ideal Joule-Brayton cycle



Efficiency of the ideal cycle

- The efficiency of the ideal cycle is:

$$\eta_{id} = 1 - \frac{Q_2}{Q_1} \quad \text{dove} \quad Q_1 = c_p\left(T_3 - T_2\right) \ \text{ e } \ Q_2 = c_p\left(T_4 - T_1\right)$$

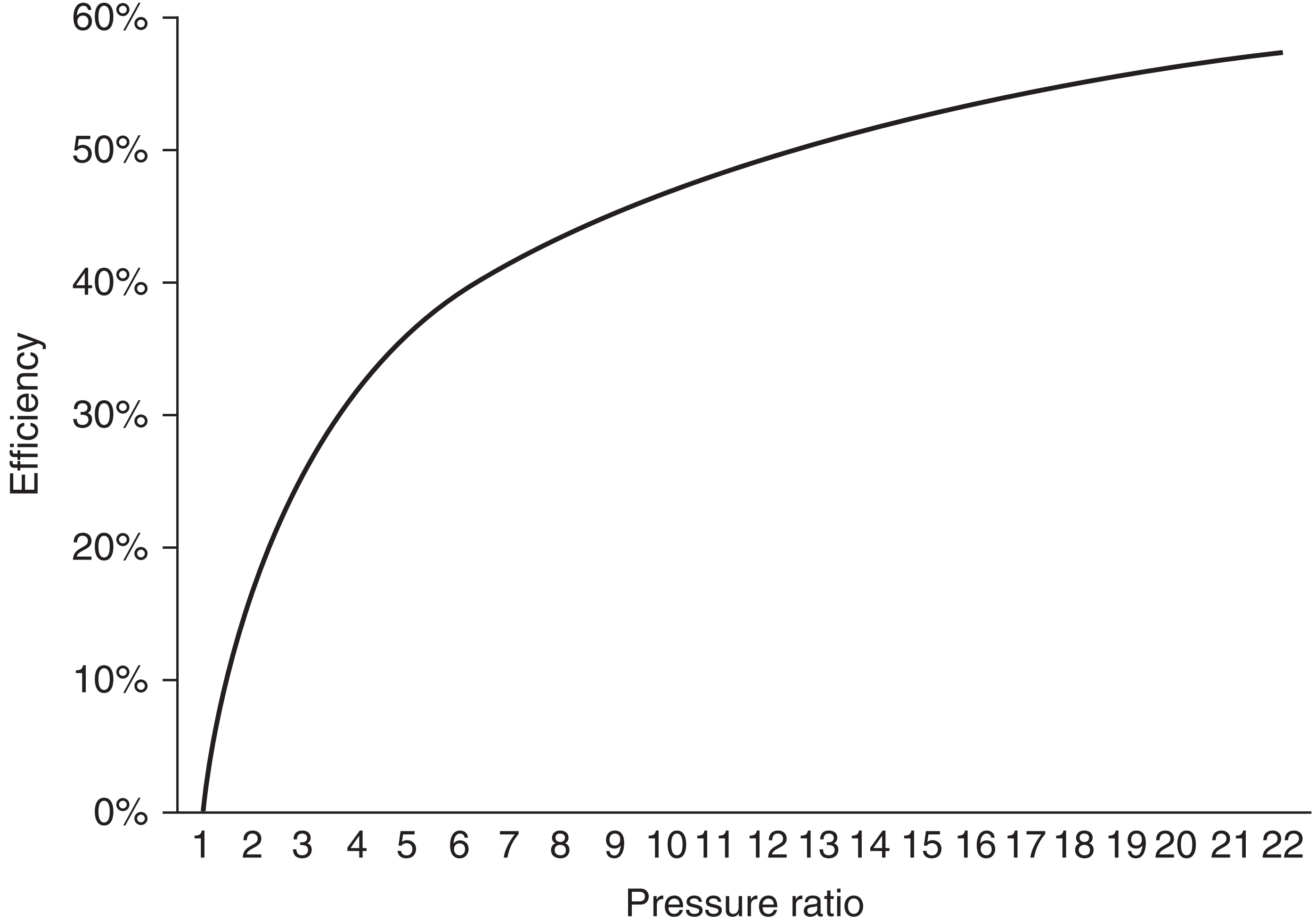
- By assuming c_p=costant:

$$\eta_{id} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2}\frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_2} - 1} = 1 - \frac{T_1}{T_2}\frac{\frac{T_4}{T_3}\frac{T_3}{T_1} - 1}{\frac{T_1}{T_2}\frac{T_3}{T_1} - 1}$$

- And assuming isentropic and adiabatic compression and expansion:

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{p_2}{p_1}^{\frac{k-1}{k}} = \beta^{\varepsilon} \qquad \Rightarrow \quad \eta_{id} = 1 - \frac{1}{\beta^{\varepsilon}}$$

Efficiency of the ideal cycle



Ideal cycle specific work

- The ideal cycle specific work is:

$$L_{id} = Q_1 - Q_2 = c_p\left(T_3 - T_2\right) - c_p\left(T_4 - T_1\right)$$

- By assuming c_p=costant:

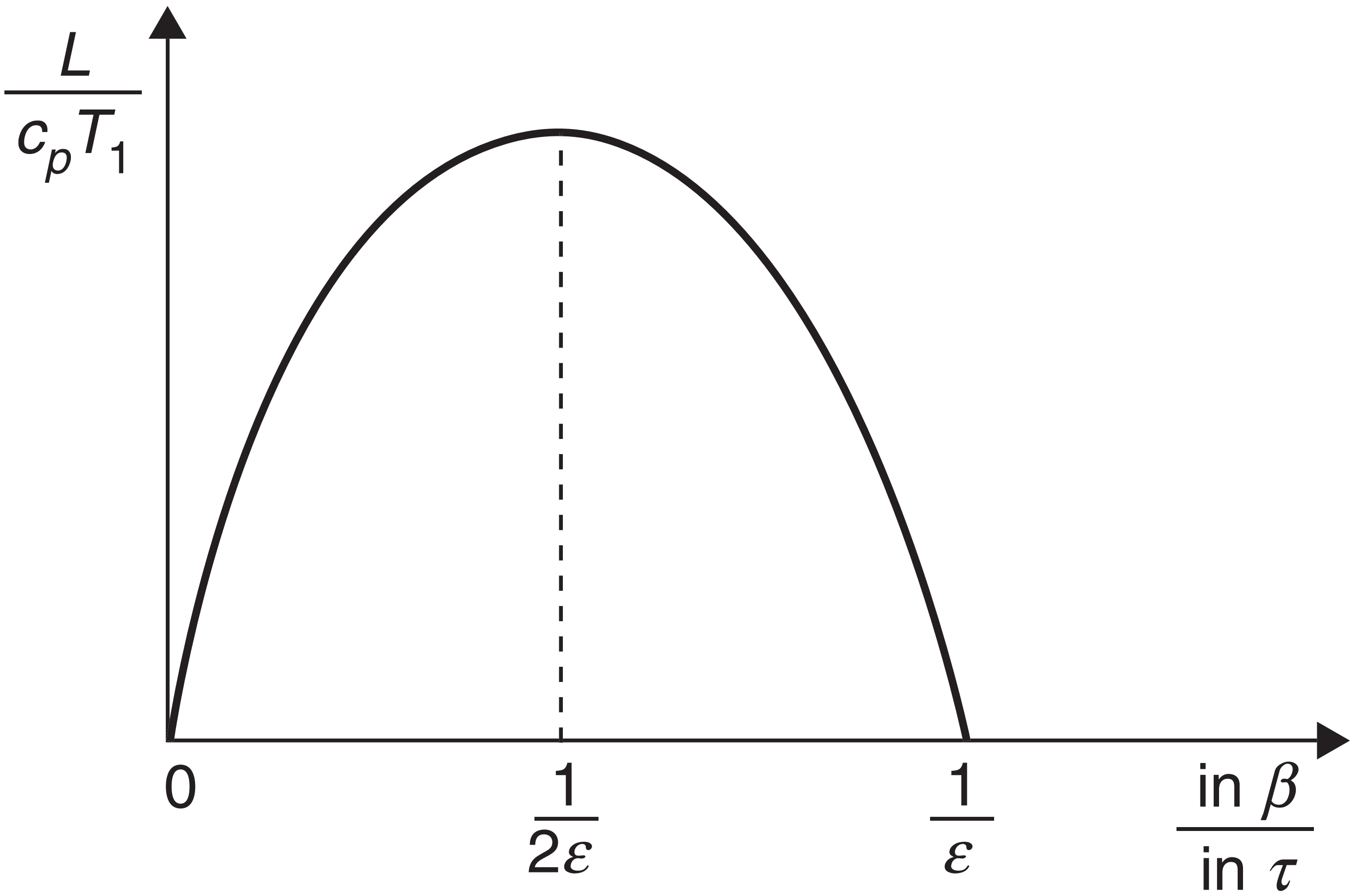
$$\frac{L_{id}}{c_p T_1} = \frac{T_3}{T_1} - \frac{T_2}{T_1} - \frac{T_4}{T_1} + 1 = \tau - \beta^{\varepsilon} - \frac{\tau}{\beta^{\varepsilon}} + 1$$

- Since $\beta > 1$ and $\beta^{\varepsilon} < \frac{T_3}{T_1}$

It is possible to calculate the pressure ratio at which the specific work is maximum:

$$\frac{d}{d\beta}\left(\frac{L_{id}}{c_p T_1}\right) = -\varepsilon\beta^{\varepsilon-1} + \tau\,\varepsilon\,\beta^{-\varepsilon-1} = 0 \qquad \Rightarrow \qquad \beta = \tau^{1/2\varepsilon}$$

Ideal specific work



Limit cycle efficiency

- The limit cycle (real fluid in ideal components) efficiency is: $\eta_l = 1 - \dfrac{Q_2}{Q_1}$
- where

$$Q_1 = \int_2^3 c_p\,dT = c_{p23}\left(T_3 - T_2\right) \qquad Q_2 = \int_1^4 c_p\,dT = c_{p14}\left(T_4 - T_1\right)$$

- By using: $\gamma = c_{p14} / c_{p23}$

$$\eta_l = 1 - \gamma \frac{T_4 - T_1}{T_3 - T_2} = 1 - \gamma \frac{T_1}{T_2} \frac{\dfrac{T_4}{T_1} - 1}{\dfrac{T_3}{T_2} - 1} = 1 - \gamma \frac{T_1}{T_2} \frac{\dfrac{T_4}{T_3}\dfrac{T_3}{T_1} - 1}{\dfrac{T_1}{T_2}\dfrac{T_3}{T_1} - 1}$$

- Since c_p is a function of temperature:

$$\frac{T_2}{T_1} = \beta^{\varepsilon_c} \qquad \frac{T_3}{T_4} = \beta^{\varepsilon_e} \qquad \Rightarrow \quad \eta_l = 1 - \frac{\gamma}{\beta^{\varepsilon_c}} \frac{\tau / \beta^{\varepsilon_e} - 1}{\tau / \beta^{\varepsilon_c} - 1}$$

Influence of the mass flowrate

- In open Joule-Brayton cycles the flowrate in the compressor and turbine is different. This is mainly due to the addition of the fuel flow rate in the combustion chamber.
- We can use a paramter that defines the air/fuel ratio:

$$\alpha = Air\ mass\ flowrate / Fuel\ mass\ flowrate$$

- By assuming a perfect combustion of a fuel with heating value (H), we can write: $Q_1 = \dfrac{H_i}{\alpha + 1}$

Influence of the mass flowrate

- We can define excess of air: $\lambda = \dfrac{\alpha - \alpha_{st}}{\alpha_{st}}$
- And we can calculate the specific work referred to the compressor's inlet air flowrate:

$$\frac{L}{c_p T_1} = \frac{(\alpha+1)}{\alpha}\left(\frac{T_3}{T_1} - \frac{T_4}{T_1}\right) - \left(\frac{T_2}{T_1} - 1\right) = \frac{(\alpha+1)}{\alpha}\left(\tau - \frac{\tau}{\beta^{\varepsilon}}\right) - \beta^{\varepsilon} + 1$$

- If we substract the specific work for the case without change in mass flowrate:

$$\frac{L}{c_p T_1} = \left(\tau - \frac{\tau}{\beta^{\varepsilon}}\right) - \beta^{\varepsilon} + 1$$

- We obtain:

$$\frac{(\alpha+1)}{\alpha}\left(\tau - \frac{\tau}{\beta^{\varepsilon}}\right) - \beta^{\varepsilon} + 1 - \left(\tau - \frac{\tau}{\beta^{\varepsilon}}\right) - \beta^{\varepsilon} + 1 = \left(\frac{(\alpha+1)}{\alpha} - 1\right)\left(\tau - \frac{\tau}{\beta^{\varepsilon}}\right) = \frac{\tau}{\alpha}\left(1 - \frac{1}{\beta^{\varepsilon}}\right)$$

Influence of the mass flowrate

- As far as the efficiency is concerned:

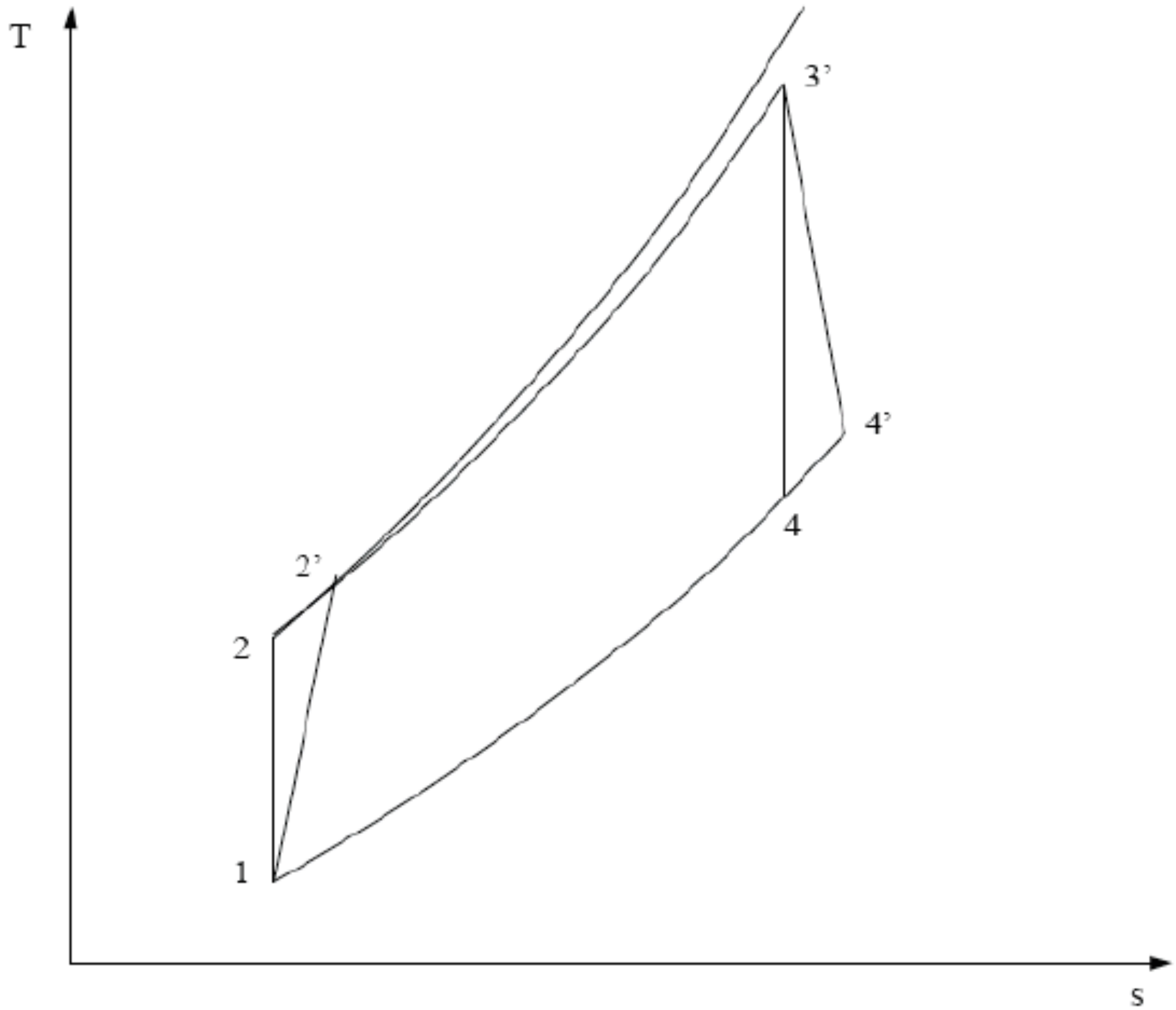
$$\eta_l = \frac{L_T - L_C}{Q_1} = \frac{(\alpha+1)c_p\left(T_3 - T_4\right) - \alpha c_p\left(T_2 - T_1\right)}{c_p\left[(\alpha+1)T_3 - \alpha T_2\right]} =$$

$$= \frac{\left[(\alpha+1)T_3 - \alpha T_2\right] - (\alpha+1)T_4 + \alpha T_1}{\left[(\alpha+1)T_3 - \alpha T_2\right]} = 1 - \frac{(\alpha+1)T_4 - \alpha T_1}{(\alpha+1)T_3 - \alpha T_2} =$$

$$= 1 - \frac{T_1}{T_2}\frac{(\alpha+1)\tau/\beta^{\varepsilon} - \alpha}{(\alpha+1)\tau/\beta^{\varepsilon} - \alpha} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{\beta^{\varepsilon}}$$

We can say that the efficiency is not affected by a change in the mass flowrate

Real gas turbine cycle



Real cycle efficiency

- The real cycle efficiency can be defined as:

$$\eta_r = \frac{L_{Tr} - L_{Cr}}{Q_{1r}}$$

- The limit cycle efficiency is:

$$\eta_l = \frac{L_T - L_C}{Q_1}$$

- We can define the internal efficiency:

$$\eta_i = \frac{\eta_r}{\eta_l}$$

- So that the real cycle efficiency can be written as:

$$\eta_r = \left(1 - \frac{1}{\beta^{\varepsilon}}\right)\eta_i$$

Real cycle efficiency

- The internal efficiency can be written as:

$$\eta_i = \frac{\eta_r}{\eta_l} = \frac{Q_1}{Q_{1r}} \frac{L_{Tr} - L_{Cr}}{L_T - L_C} = \vartheta \frac{L_{Tr} - L_{Cr}}{L_T - L_C} = \vartheta \frac{\eta_T L_T - \frac{L_C}{\eta_C}}{L_T - L_C} = \frac{\vartheta}{\eta_C} \frac{\eta_C \eta_T - \frac{L_C}{L_T}}{1 - \frac{L_C}{L_T}}$$

$$= \frac{\vartheta}{\eta_C} \frac{1 - \eta_C \eta_T + 1 - \frac{L_C}{L_T}}{1 - \frac{L_C}{L_T}} = \frac{\vartheta}{\eta_C} \left(1 - \frac{1 - \eta_C \eta_T}{1 - \frac{L_C}{L_T}} \right)$$

- The ratio of the compressor's over the turbine's work is:

$$\frac{L_C}{L_T} = \frac{c_p (T_2 - T_1)}{c_p (T_3 - T_4)} \cong \frac{(T_2 - T_1)}{(T_3 - T_4)} = \frac{T_2}{T_3} \frac{1 - \frac{T_1}{T_2}}{1 - \frac{T_4}{T_3}} = \frac{T_2}{T_3} = \frac{T_1}{T_3} \frac{T_2}{T_1} = \frac{\beta^\varepsilon}{\tau}$$

Real cycle efficiency

- Thus:

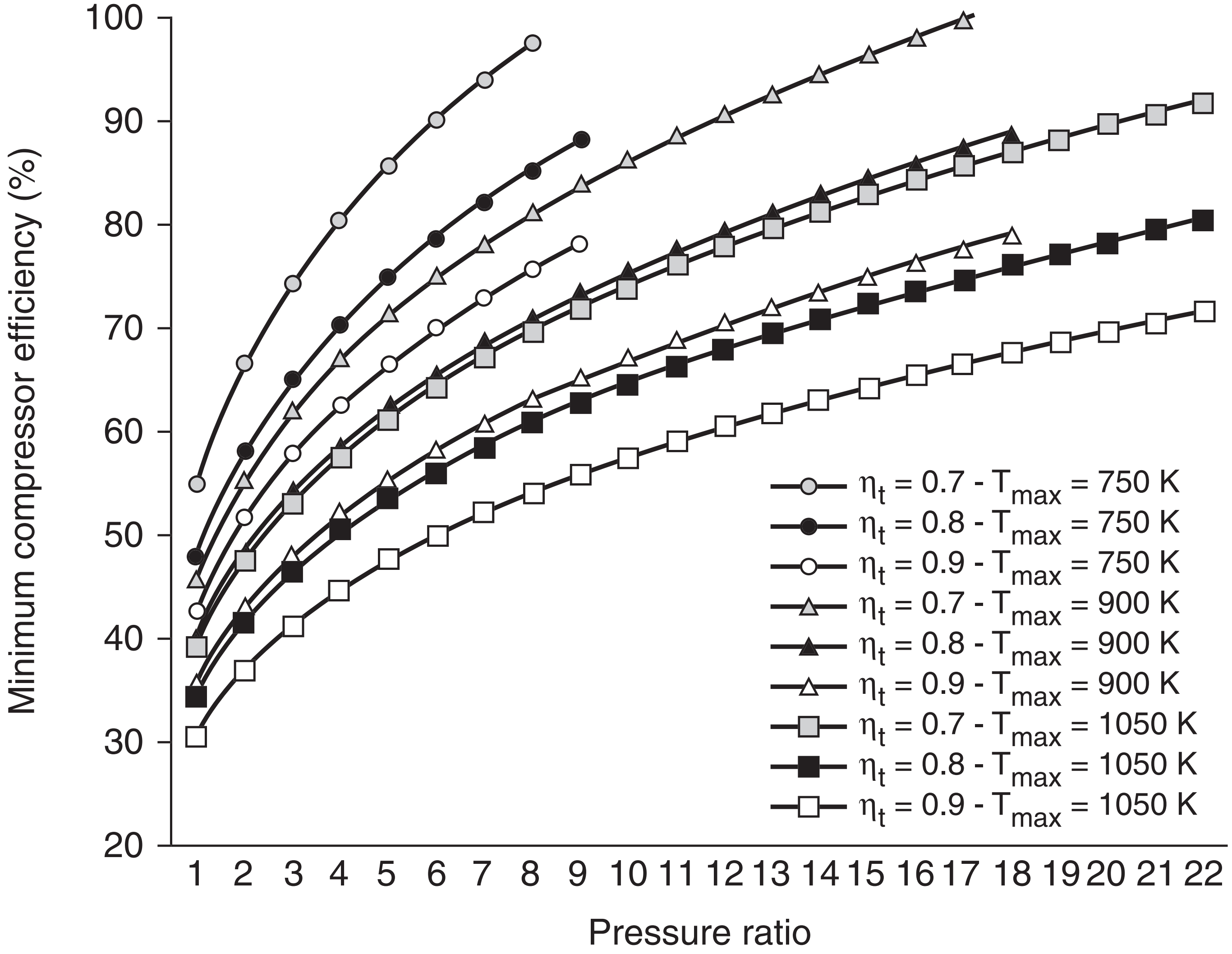
$$\eta_i = \frac{\vartheta}{\eta_C}\left(1 - \frac{1 - \eta_C \eta_T}{1 - \dfrac{\beta^{\varepsilon}}{\tau}}\right)$$

- And eventually:

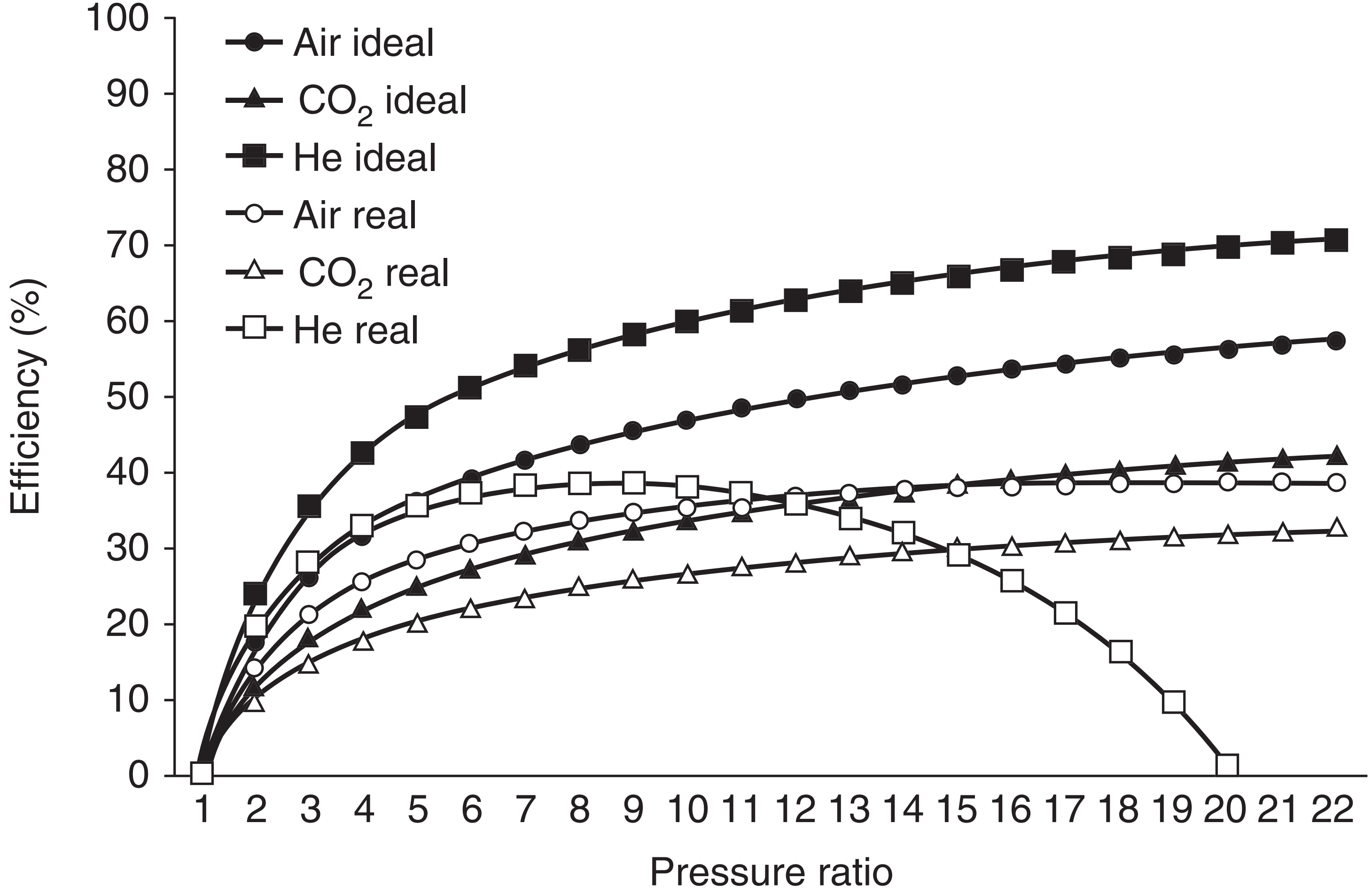
$$\eta_r = \eta_l \eta_i = \left(1 - \frac{1}{\beta^{\varepsilon}}\right)\frac{\vartheta}{\eta_C}\left(1 - \frac{1 - \eta_C \eta_T}{1 - \dfrac{\beta^{\varepsilon}}{\tau}}\right)$$

- Which is positive if: $\eta_C \eta_T > \dfrac{\beta^{\varepsilon}}{\tau}$

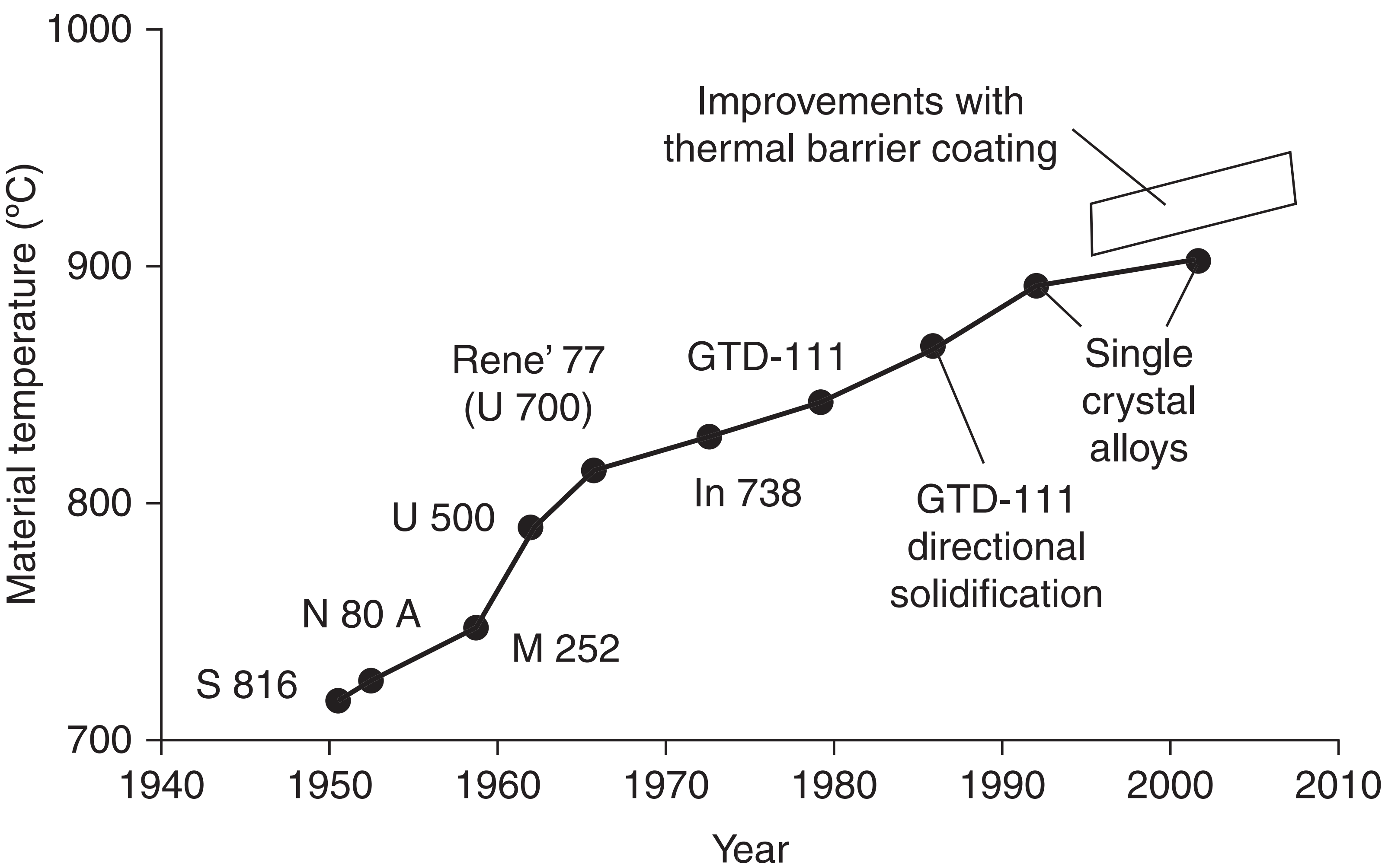
Limits of the performance



Working fluid

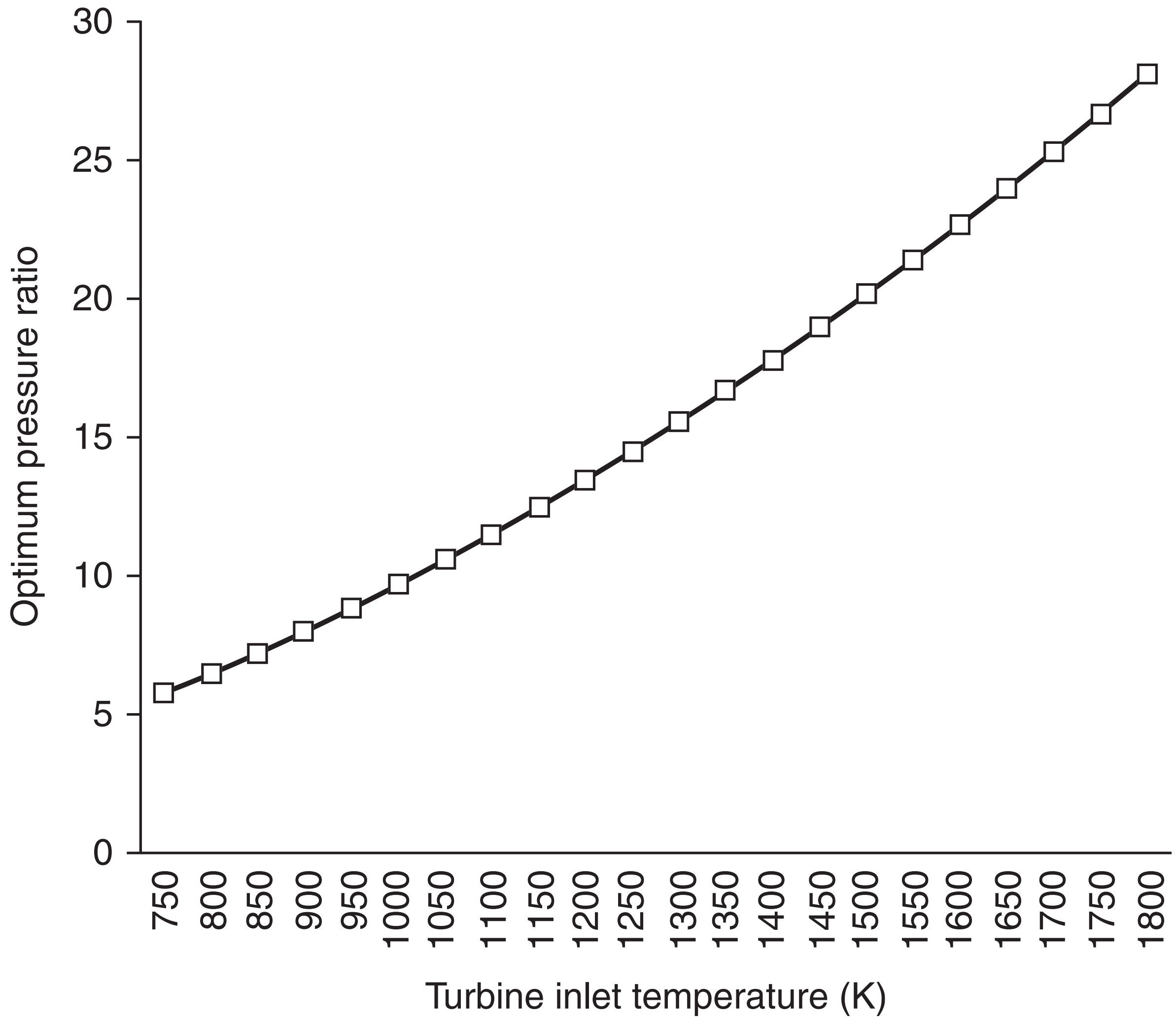


Trend of TIT

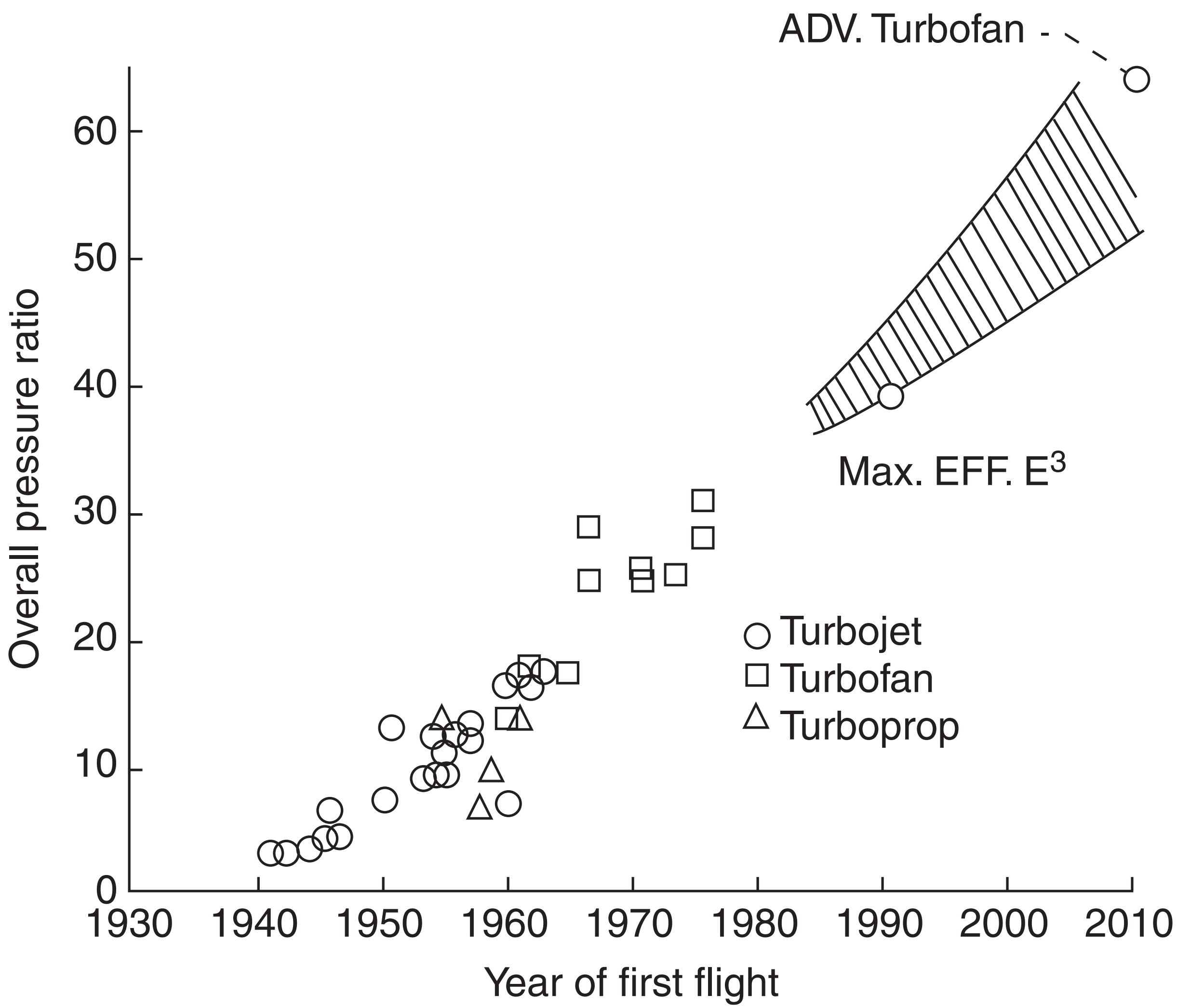


1000
900
800
700
Material temperature (ºC)
1940
1950
1960
1970
1980
1990
2000
2010
Year
Improvements with
thermal barrier coating
Single
crystal
alloys
GTD-111
directional
solidification
GTD-111
In 738
Rene' 77
(U 700)
U 500
M 252
N 80 A
S 816

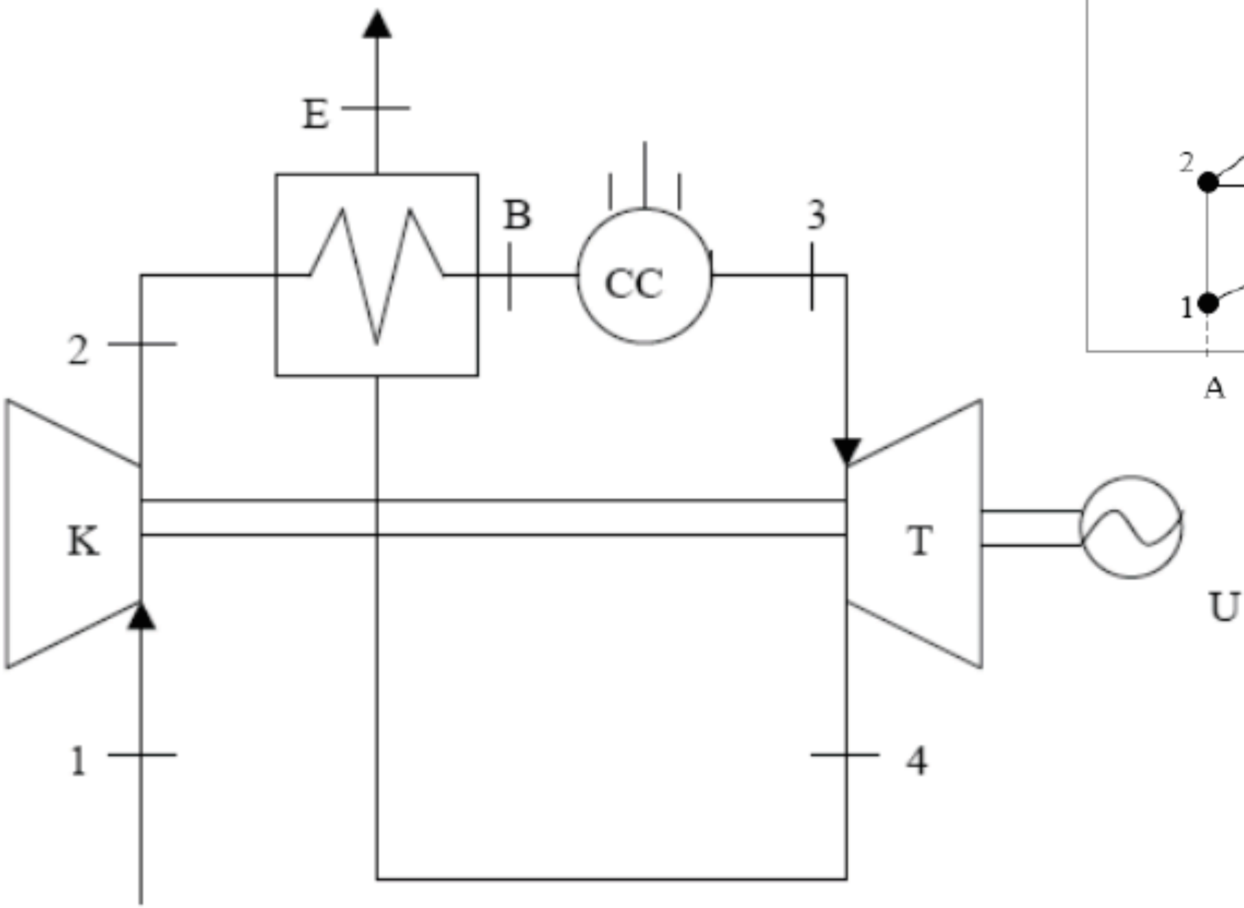
Pressure ratio at maximum power

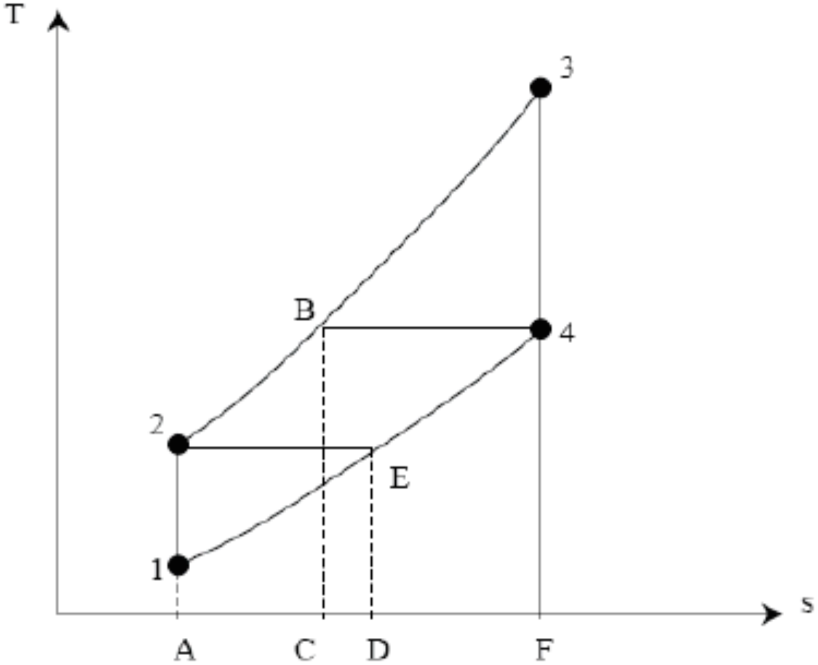


Trend of pressure ratio



Recuperated gas turbine





$$T_4 > T_2$$

Efficiency

- Let us define the recuperation ratio and set it as 1: $R = \frac{T_B - T_2}{T_4 - T_2} = 1 \quad i.e. \quad T_B = T_4$
- We can calculate the efficiency as:

$$\eta_{rec} = 1 - \frac{Q_{2rec}}{Q_{1rec}} = 1 - \frac{T_E - T_1}{T_3 - T_B} = 1 - \frac{T_2 - T_1}{T_3 - T_4} = 1 - \frac{\beta^\varepsilon - 1}{\tau - \frac{\tau}{\beta^\varepsilon}} = 1 - \frac{\beta^\varepsilon}{\tau}$$

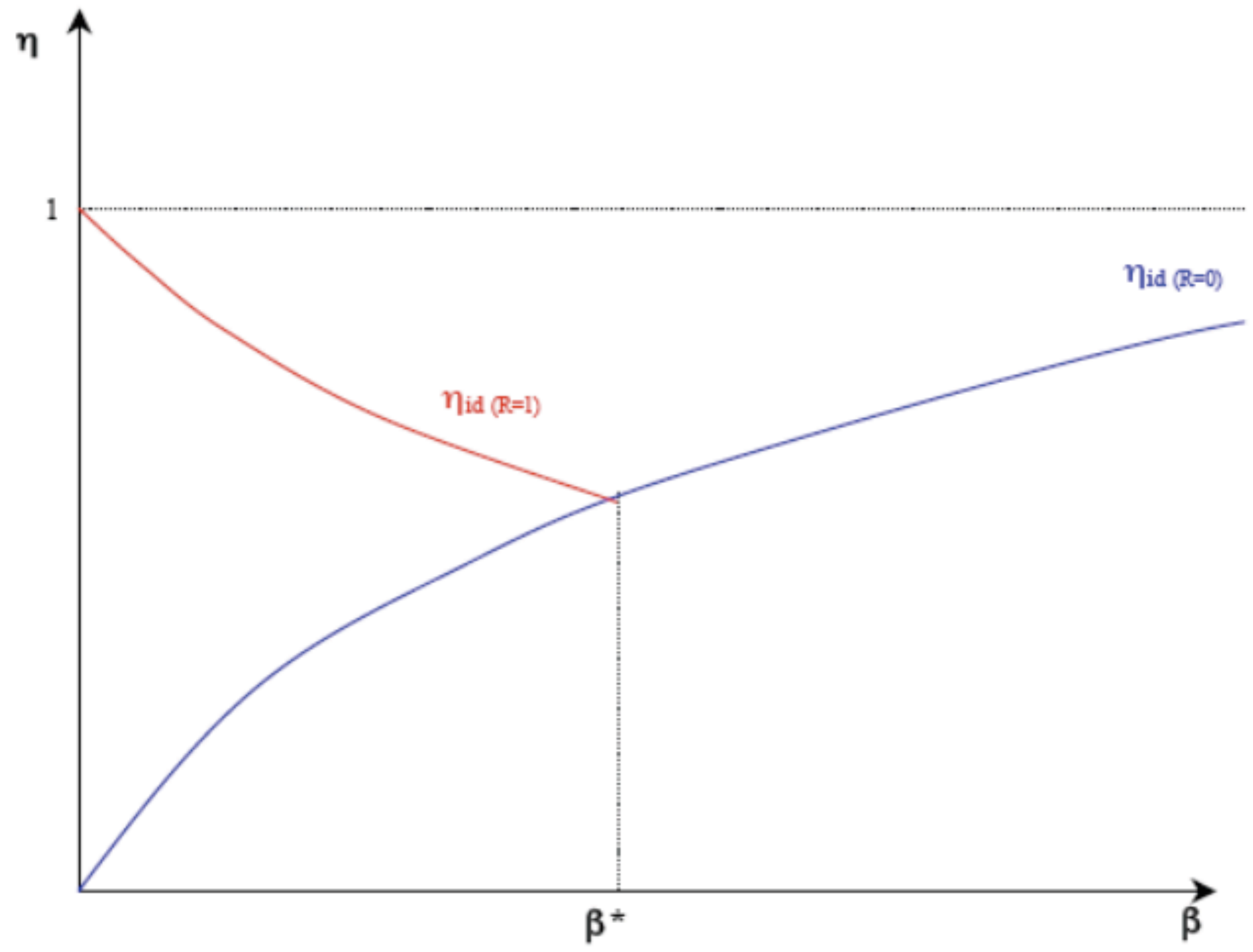
- The point where the efficiency is the same as that of the simple cycle is given by:

$$1 - \frac{\beta^\varepsilon}{\tau} = 1 - \frac{1}{\beta^\varepsilon} \Rightarrow \tau = \beta^{2\varepsilon} \Rightarrow \beta = \tau^{1/2\varepsilon}$$

- A similar expression can be derived from

$$T_2 = T_4 \Rightarrow \frac{T_4}{T_3}\frac{T_3}{T_1} = \frac{T_2}{T_1} \Rightarrow \frac{\tau}{\beta^\varepsilon} = \beta \Rightarrow \beta = \tau^{1/2}$$

Recuperated gas turbine

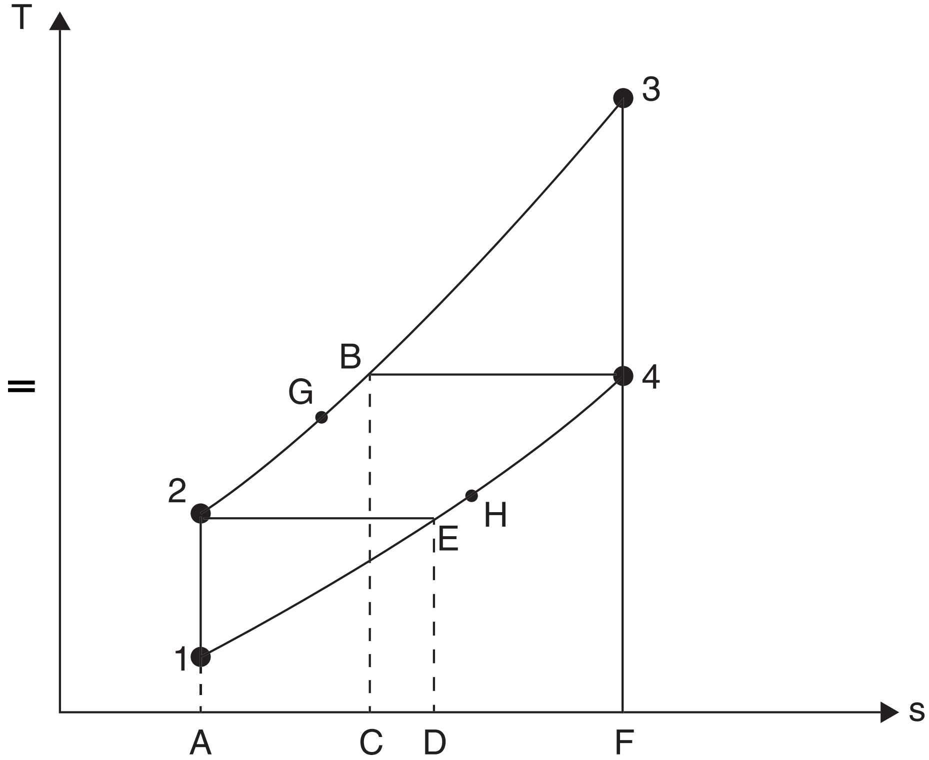


Recuperated gas turbines with R≠1

- If R<1 we can write: $R=\frac{T_G-T_2}{T_4-T_2}$

$$Q_{1rec}=c_p\left(T_3-T_4\right)+\left(1-R\right)c_p\left(T_4-T_2\right)$$

$$Q_{2rec}=c_p\left(T_2-T_1\right)+\left(1-R\right)c_p\left(T_4-T_2\right)$$



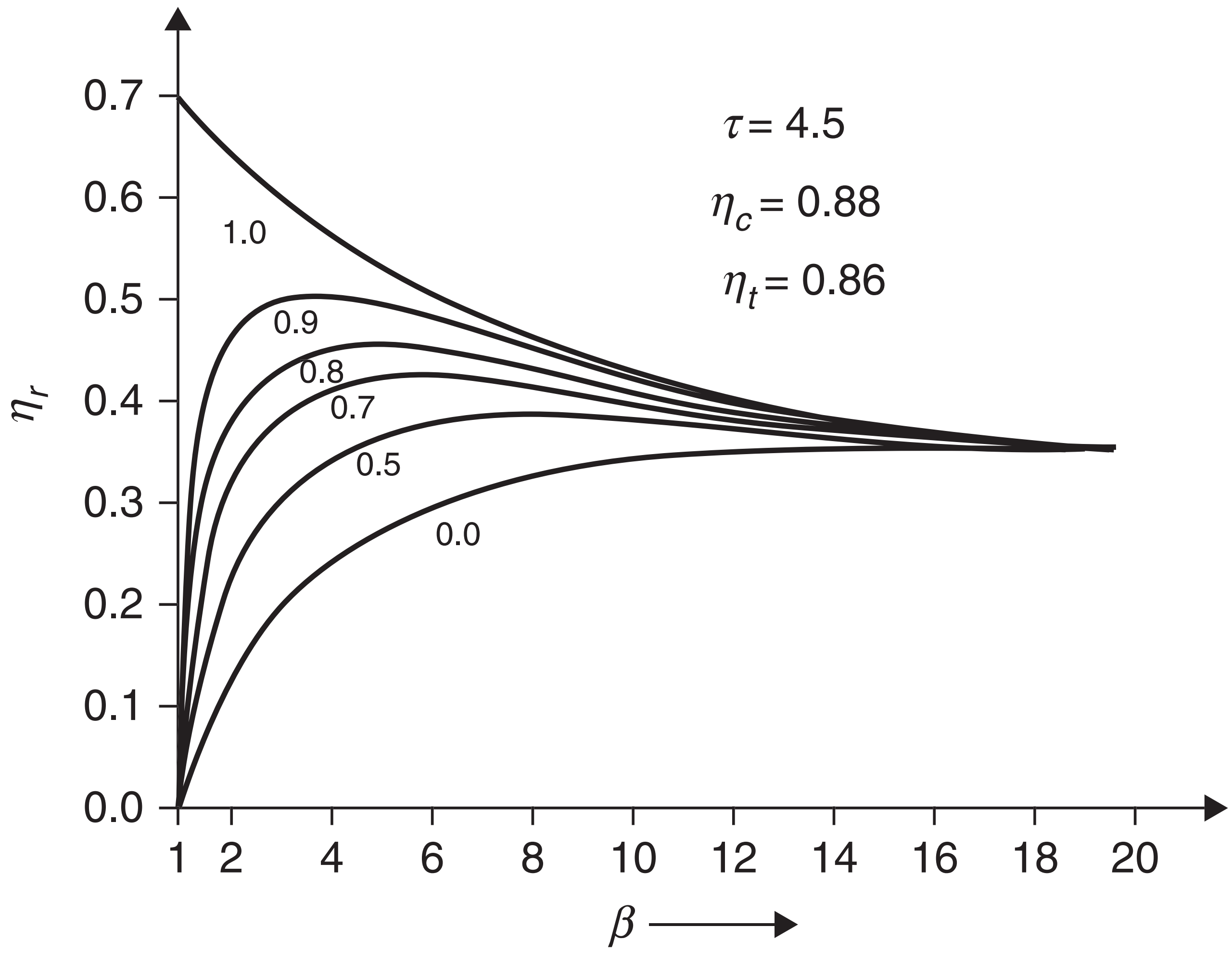
- From which:

$$\eta_{recR}=1-\frac{c_p\left(T_2-T_1\right)+\left(1-R\right)c_p\left(T_4-T_2\right)}{c_p\left(T_3-T_4\right)+\left(1-R\right)c_p\left(T_4-T_2\right)}=$$

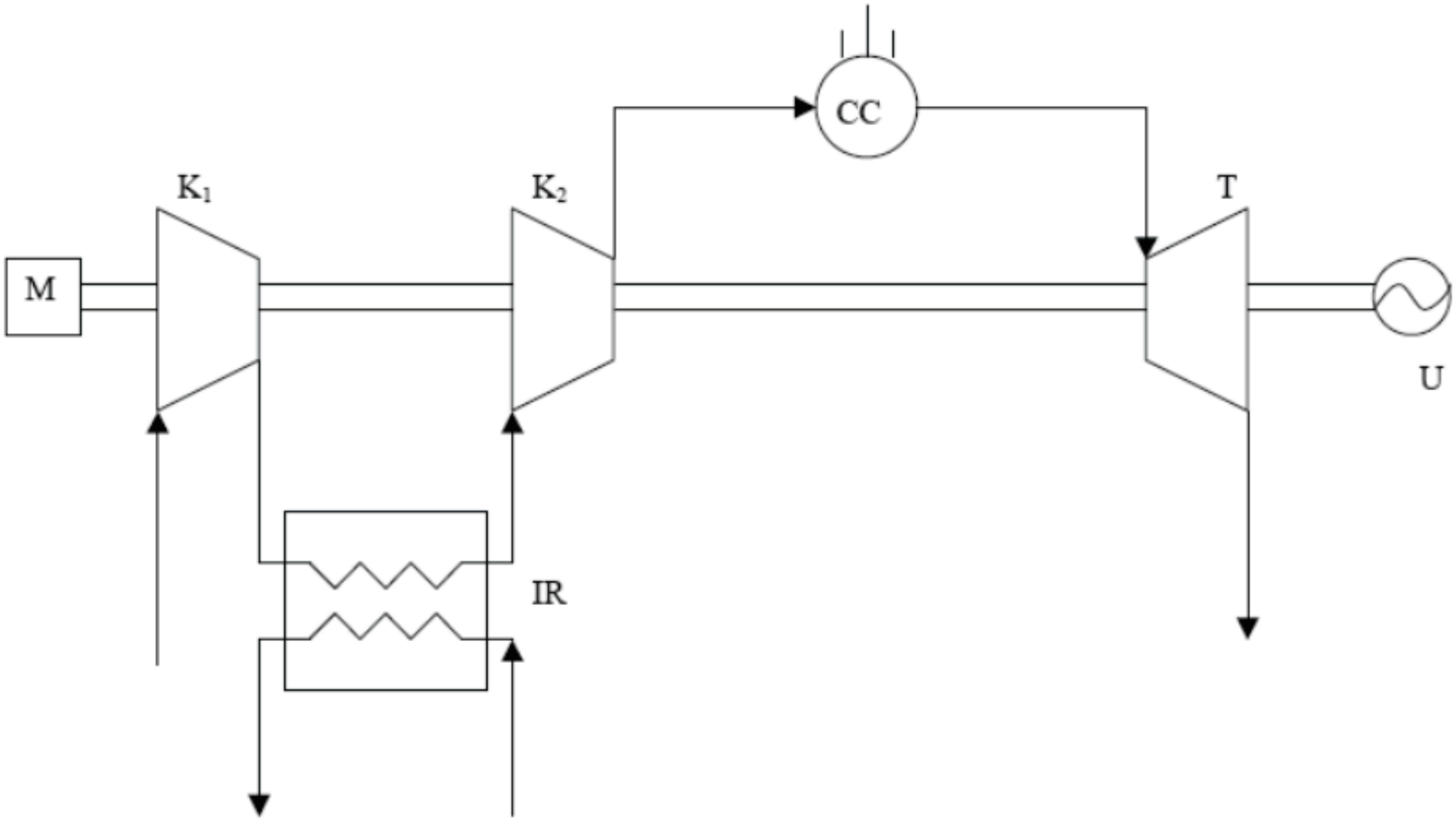
-

$$=1-\frac{\tau-\dfrac{\tau}{\beta^{\varepsilon}}+\left(1-R\right)\left(\dfrac{\tau}{\beta^{\varepsilon}}-\beta^{\varepsilon}\right)}{\beta^{\varepsilon}-1+\left(1-R\right)\left(\dfrac{\tau}{\beta^{\varepsilon}}-\beta^{\varepsilon}\right)}$$

Recuperated gas turbines with R≠1



Intercooled compression

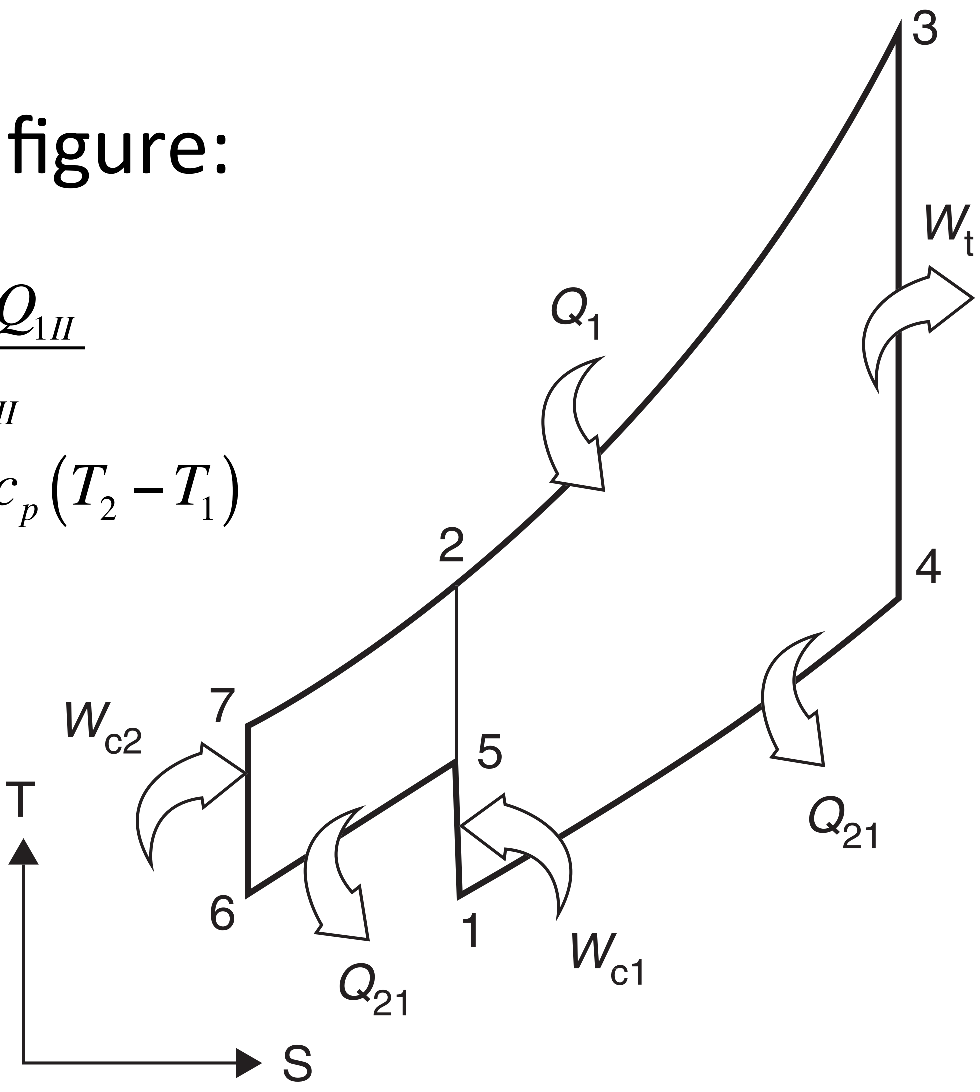


Intercooled compression

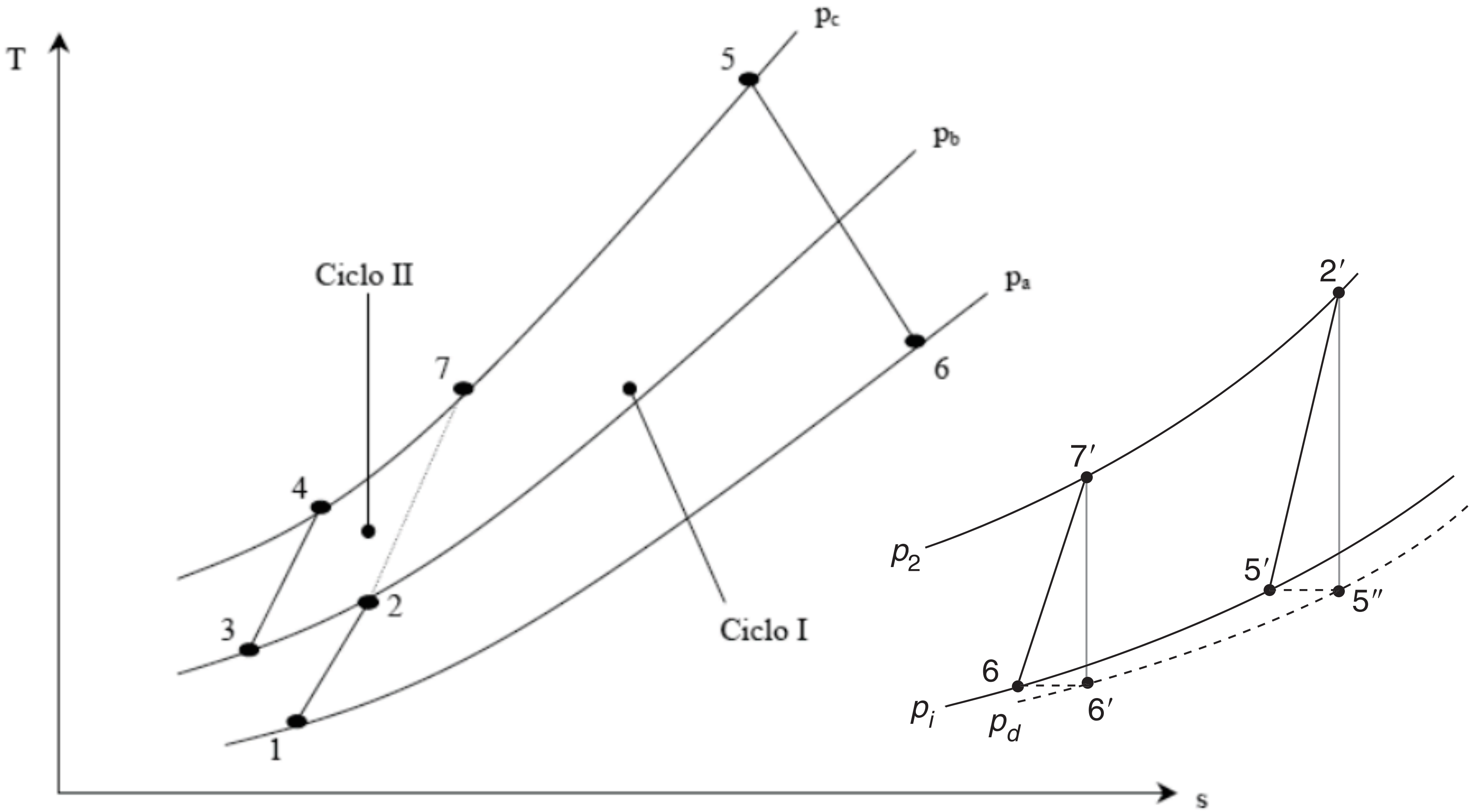
- If we refer to the figure:

$$\eta_{IC} = \frac{W_I + W_{II}}{Q_{1I} + Q_{1II}} = \frac{\eta_I Q_{1I} + \eta_{II} Q_{1II}}{Q_{1I} + Q_{1II}}$$

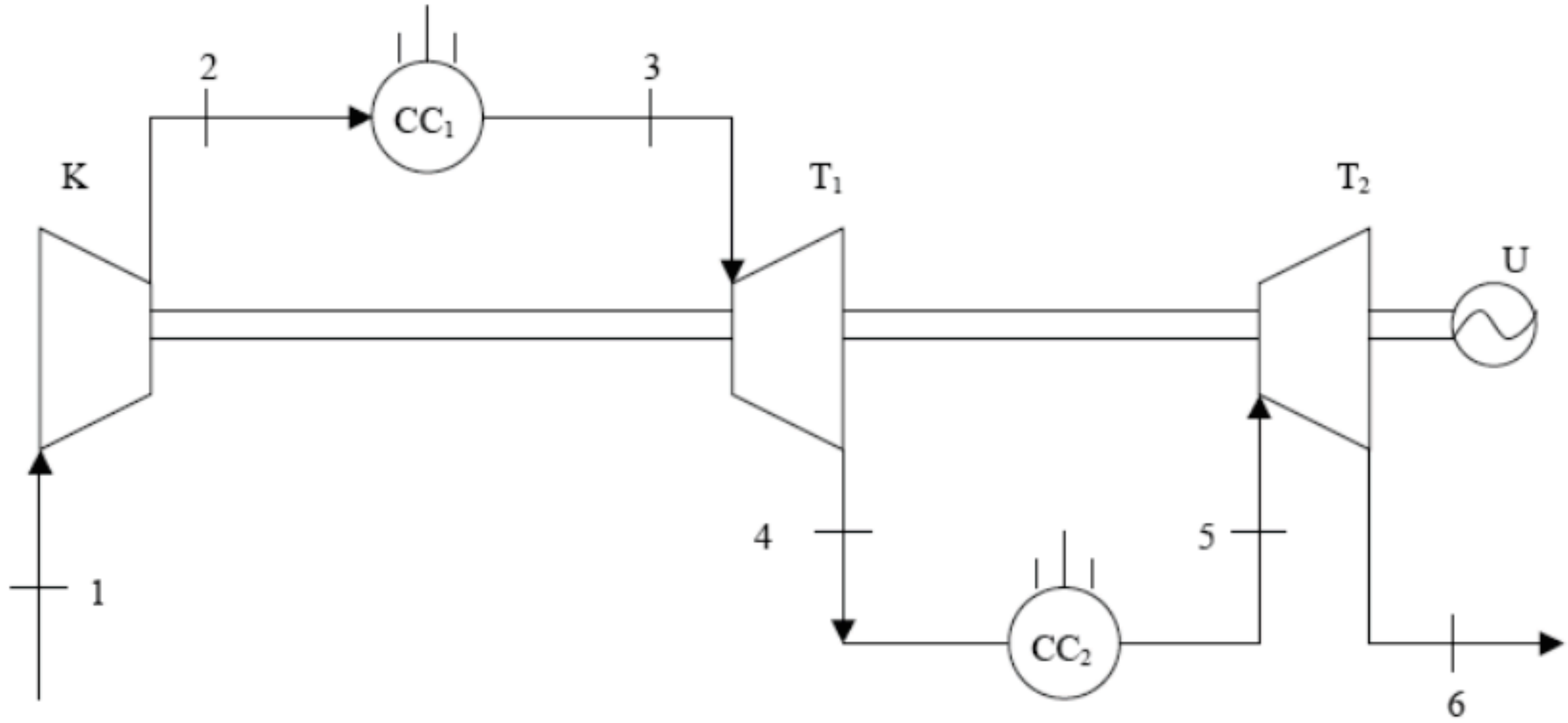
$$W_{IC} = c_p \left(T_5 - T_1 + T_7 - T_6 \right) < c_p \left(T_2 - T_1 \right)$$



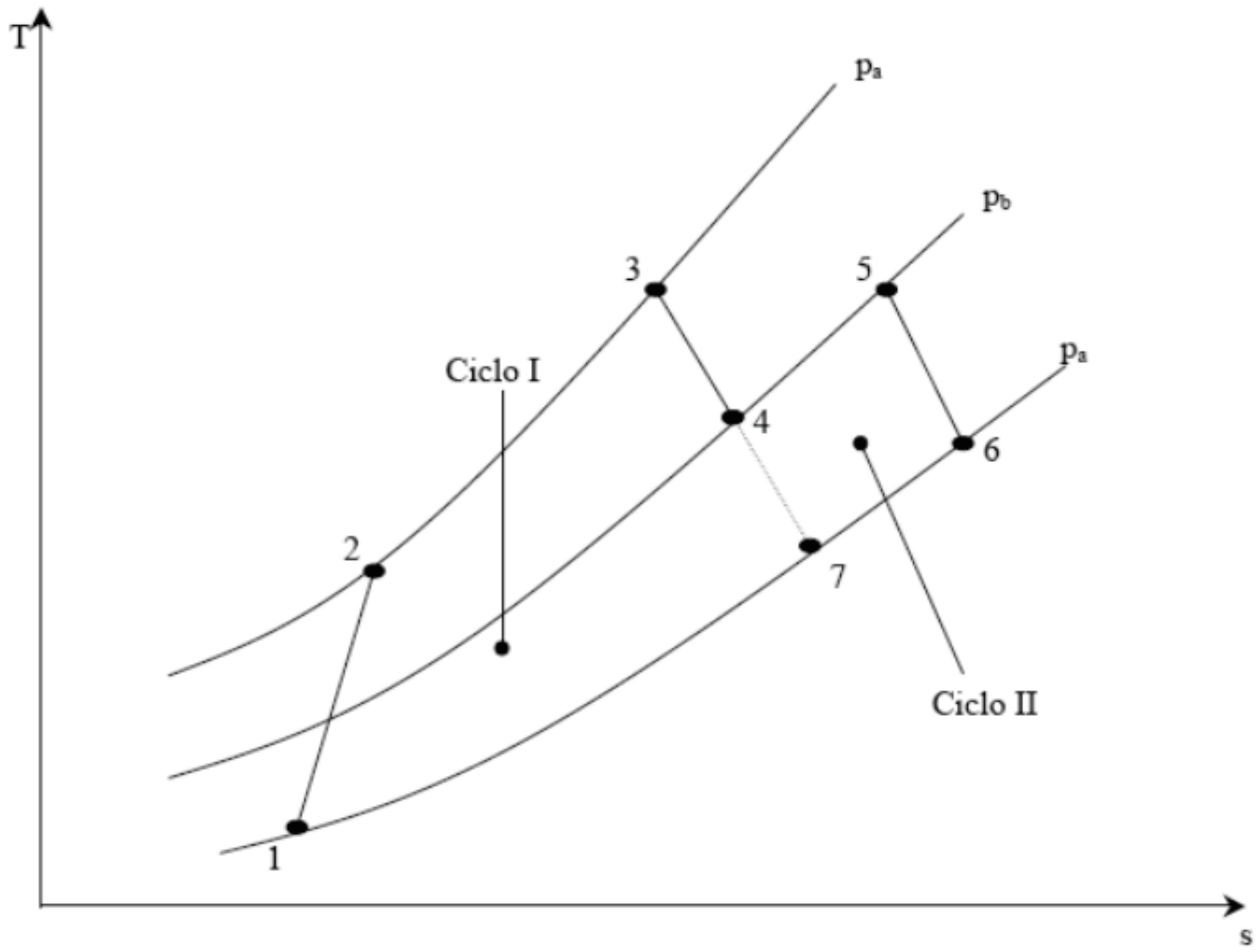
Intercooled compression



Reheat gas turbine



Reheat gas turbine



Steam injected gas turbine