①

$$f(x) = \ln(x+1) \qquad x > -1$$

○ Condizionamento.

$$\epsilon_m = \frac{f'(x)}{f(x)} x \cdot \epsilon_x \qquad |\epsilon_x| \leq u$$

$$C_x = \frac{f'(x)}{f(x)} x$$

$$|C_x| = \left| \frac{\frac{1}{x+1}}{\ln(x+1)} x \right| = \left| \frac{x}{x+1} \frac{1}{\ln(x+1)} \right|$$

$$|C_x| = \frac{|x|}{|x+1| \, |\ln(x+1)|}$$

Problema malcondizionato quando

$$x \rightarrow -1^+$$

1

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Problema malcondizionato quando

$$x \to -1^+$$

2

$x \to -1$

$$\lim_{x \to -1^+} \left| (x+1) \ln (x+1) \right|$$

$$= \lim_{t \to 0^+} t \cdot \ln t \Rightarrow \lim_{t \to 0^+} \frac{\ln t}{\frac{1}{t}}$$

$$= \lim_{t \to 0^+} \frac{\frac{1}{t}}{-\frac{1}{t^2}} = \lim_{t \to 0^+} -\frac{t^2}{t} = \lim_{t \to 0^+} -t = 0$$

MAL CONDIZIONATE $\quad x = 0$?

$$\lim_{x \to 0} \frac{x}{\ln(x+1)} =$$

$$\lim_{x \to 0} \frac{1}{\frac{1}{x+1}} = \lim_{x \to 0} x+1 = 1$$

NO

$$\mathrm{lN}(x) = \ln(x)(1+\epsilon) \qquad |\epsilon| \le u$$

Stabilità

$$\begin{tikzcd}
x \arrow[dr, "\frac{x}{x+1}"] & & \\
 & \overset{\epsilon_1}{x+1} \arrow[r, "\frac{1}{\ln(x+1)}"] & \overset{\epsilon_2}{\ln(x+1)} \\
1 \arrow[ur, "\frac{1}{x+1}"'] & &
\end{tikzcd}$$

$$|\epsilon_i| \le u$$

$$t \longrightarrow \ln(t)$$

$$\frac{f'(t)}{f(t)} \cdot t = \frac{\frac{1}{t}}{\ln(t)} \cdot t$$

$$= \frac{1}{\ln(t)}$$

$$|\epsilon_{alg}| = \left|\epsilon_2 + \frac{1}{\ln(x+1)}\epsilon_1\right| \le |\epsilon_2| + \frac{1}{|\ln(x+1)|}|\epsilon_1|$$

$$\le u\left(1 + \frac{1}{|\ln(x+1)|}\right)$$

4

Numericamente instabile.

Se $\left|\ln(x+1)\right| \simeq 0 \Rightarrow x \simeq 0$

$$\frac{1}{a(1+a)} = \frac{1}{a} - \frac{1}{1+a} \qquad a \neq 0, -1$$

- Si studi il condizionamento
- Si studi lo stabilità.

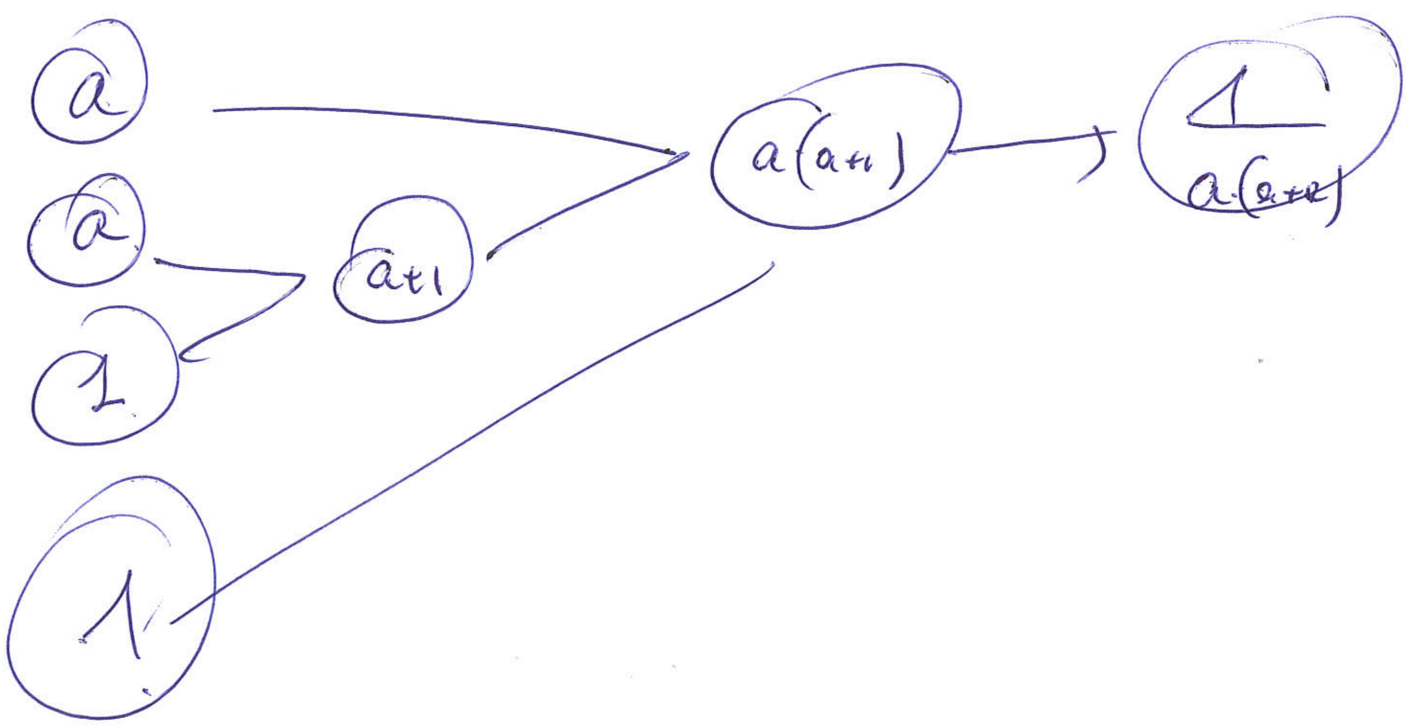
$$\epsilon_{m} = \frac{f'(a)}{f(a)}\, a\, \epsilon_a$$

$$C_a = \frac{f'(a)}{f(a)}\, a = \frac{-\frac{1}{a^2} + \frac{1}{(1+a)^2}}{\frac{1}{a(1+a)}}\, a$$

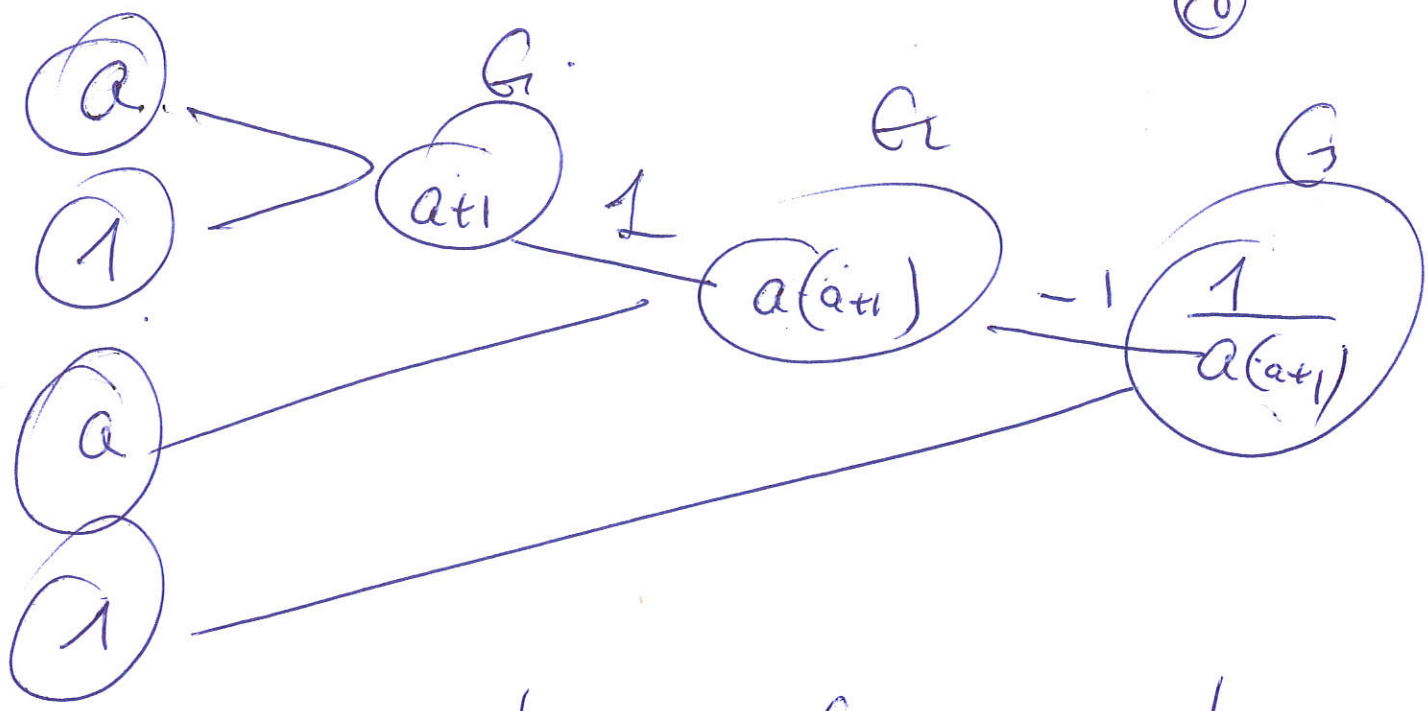
$$C_a = \frac{(a+1)^2 - a^2}{\cancel{a^2}\,\cancel{(1+a)^2}} \cdot \cancel{a^2} \cdot \cancel{(1+a)}$$

$$|C_a| = \left| \frac{1+2a}{1+a} \right|$$

Problemi nel condiz. per $a \simeq -1$



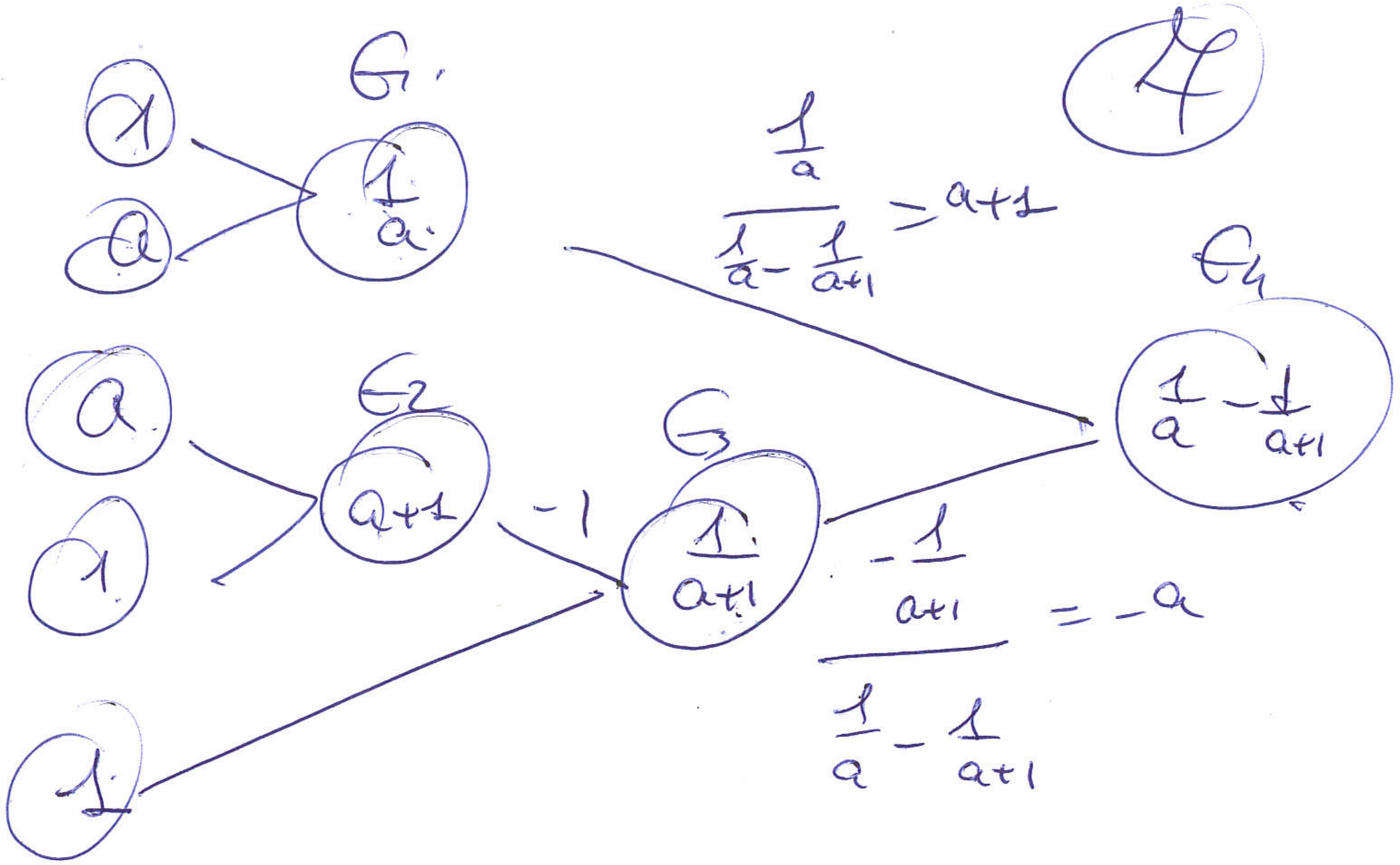
8



$$|G_{delg}| = |G_3 - (G_2 + G_1)|$$

$$\leq |G_3| + |G_2| + |G_1| \leq 3u$$

1 Algoritm 1 Numerical Stable



$$\frac{\frac{1}{a}}{\frac{1}{a}-\frac{1}{a+1}} = \frac{\frac{1}{a}}{\frac{a+1-a}{a(a+1)}} = \frac{1}{\cancel{a}}\cdot\cancel{a}\cdot(a+1)$$

$$\frac{-\frac{1}{a+1}}{\frac{1}{a}-\frac{1}{a+1}} = \frac{-\frac{1}{a+1}}{\frac{a+1-a}{a(a+1)}} = -a$$

8

$$\epsilon_{alg} = \epsilon_4 + (a+1)\,\epsilon_1$$
$$- a\,(\epsilon_3 - \epsilon_2)$$

$$= \epsilon_4 + (a+1)\,\epsilon_1 - a\cdot(\epsilon_3 - \epsilon_2)$$

$$|\epsilon_{alg}| = \left|\,\epsilon_4 + (a+1)\,\epsilon_1 - a\cdot(\epsilon_3 - \epsilon_2)\right|$$

$$\leq |\epsilon_4| + |a+1|\cdot|\epsilon_1| + |a|\cdot|\epsilon_3 - \epsilon_2|$$

$$\leq u + |a+1|\,u + |a|\cdot 2u$$

$$\leq \left(1 + |a+1| + 2|a|\right) u$$

ALGORITMO È NUMERICAMENTE INSTABILE PER $a \to \pm\infty$

$F(2,3,2,1)$

$$X = \pm 2^{p}\left(1 \cdot 2^{-1} + d_2 2^{-2} + d_3 2^{-3}\right)$$

$p \in \{-2, -1, 0, 1\}$

$d_2, d_3 \in \{0, 1\}$

$$\frac{2}{3} = 2^{p}\left(1 \cdot 2^{-1} + d_2 2^{-2} + d_3 2^{-3} + d_4 2^{-4} \ldots\right)$$

$$A = \begin{bmatrix} 1 & & \alpha \\ & \ddots \quad O & \\ & & 1 \end{bmatrix} \in R^{n \times n}$$

$$\|A\|_\infty = \max\{1, |\alpha|\}$$

$$\|A\|_1 = \max\{1, |\alpha|\}$$

$$\|A\|_2 = \sqrt{\rho(A^T A)}$$

$A^T A = B$

$\rho(A^T A) = \rho(B) = \max_{i=1 \dots n} |\lambda_i|$

$\sqrt{\rho(B)}$

$$A^T A = \begin{bmatrix} 1 & & \\ & \ddots & \\ \alpha & & 1 \end{bmatrix} \begin{bmatrix} 1 & & \alpha \\ & \ddots & \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 \cdots & \\ & & \ddots & \\ & & & 1 \\ & & & & \alpha^2 \end{bmatrix}$$

$A^T A$ diagonale. ($a_{ij} = 0$ se $i \neq j$)

gli autovalori stanno sulla diagonale principale

autovalori di $A^T A$ 1 con $n-1$ volte α^2

(11)

$$\|A\|_2 = \sqrt{\max\{1, |\alpha|^2\}}$$

$|\alpha| \le 1$ $\quad \|A\|_1 = \|A\|_\infty = 1$

$\|A\|_2 = 1$

$|\alpha| \geq 1$ $\quad \|A\|_1 = \|A\|_\infty = |\alpha|$

$\|A\|_2 = \sqrt{|\alpha|^2} = |\alpha|$

$\left(\sqrt{x^2} = |x| \right)$

α la matrice è invertibile.

$A \in \mathbb{R}^{n \times n}$ $\quad A$ è invertibile $\iff \det A \neq 0$

A è invertibile $\iff \left(Ax = 0 \iff x = 0 \right)$

A è invertibile $\iff$ $x = 0$ non è autovettore di A

(1) (12)

$\det A \rightsquigarrow$ calcolo di Laplace

$$\det A = (-1)^{n+1} \alpha \cdot \det \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} =$$

$$= (-1)^{n+1} \alpha$$

A è singolare $\iff \alpha = 0$

(2) $Ax = 0 \iff \begin{cases} \alpha x_n = 0 \\ x_1 = 0 \\ \vdots \\ x_{n-1} = 0 \end{cases}$

Se $\alpha = 0$ $\quad \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \ker A \Rightarrow A$ singolare

Se $\alpha \neq 0 \Rightarrow x_n = 0 \Rightarrow x = 0$

13

$A^{-1} \cdot A = I$

$A \cdot A^{-1} = I$

$A A^{-1} e_j = I e_j$

$$e_j = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow j\text{-esi}$$

$\boxed{A \cdot b_j = e_j}$ $j = 1 \ldots n$

le colonne dell'inversa sono soluzioni per i sistemi lin

$A \cdot b_1 = e_1$ $A \cdot b_2 = e_2$ . . . $A b_n = e_n$