RICEVIMENTO 18/07

$$A = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \\ 1 & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \boxed{\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}}$$

$$A \longrightarrow A^{(1)} = \begin{bmatrix} 1 & \cdots & 1 \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \qquad E_1 = \begin{bmatrix} 1 & & \\ -1 & \ddots & \\ \vdots & & \\ -1 & & 1 \end{bmatrix}$$

$A^{(1)} = E_1 \cdot A$ $\qquad$ $A^{(1)}$ triangolare superiore

$$A = E_1^{-1} A^{(1)} = \begin{bmatrix} 1 & & \\ 1 & \ddots & \\ 1 & & 1 \end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$

fattorizzazione LU.

$A^{(1)} = E_1 \cdot A$ $\qquad$ $A^{(1)} = \begin{pmatrix} 1 & \cdots & 1 \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$

$E_2 \qquad A^{(1)} \qquad E_2 = \begin{bmatrix} 1 & & \\ & \ddots & \\ & \vdots & \\ & x & 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & & \\ 1 & 1 & \\ 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \\ 0 & & \end{bmatrix}$$

$$A \longrightarrow B = V^{-1} A \cdot V$$

A è singolare $\Leftrightarrow$ U è singolare.

$$\cancel{\det(\lambda I - A) = \det U}$$

$$\det(\lambda I - LU) = \det(\lambda L \cdot L^{-1} - LU) =$$

$$= \det\left(L \cdot (\lambda L^{-1} - U)\right) = \underbrace{\det L}_{1} \cdot \det(\lambda L^{-1} - U)$$

$$\det(\lambda I - A) = \det(\lambda L^{-1} - U)$$

$$\det \begin{bmatrix} \lambda - 1 & -1 & \cdots & -1 \\ -\lambda & \lambda & & \\ \vdots & & \ddots & \\ -\lambda & & & \lambda \end{bmatrix}$$

$$A = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$$ simmetrico

$$A x = 0 \cdot x \qquad \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} x = 0$$

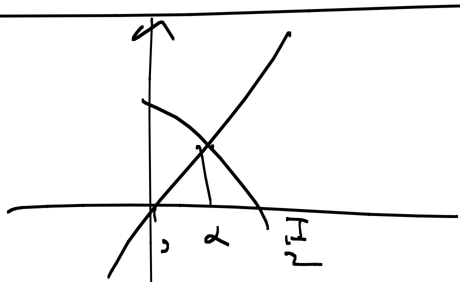
$$\begin{bmatrix} 1 \\ \vdots \\ n \end{bmatrix} \left(\sum_{i} x_i \right) = 0 \qquad X \text{ è autovett. associato a } 0$$

$$\iff \underline{\underline{\sum x_i = 0}}$$

$$V = \left\{ x \in \mathbb{R}^n : \sum_{i=0}^{n} x_i = 0 \right\}$$

$$\begin{pmatrix} x_1 \\ \cancel{x_2} \\ -x_1 - x_2 \end{pmatrix} \qquad \dim = \underline{n-1} \qquad \Gamma = n-1 \Rightarrow \Gamma = n-1$$

$$f(x) = \underline{x - \cos x = 0}$$



$$x - \cos x = 0 \iff x = \cos x \Rightarrow$$

$$\begin{cases} x_0 \in \mathbb{R} \\ x_{k+1} = g(x_k) \end{cases} \qquad g(x) = \cos x$$

$$g \in C^1([a,b]) \qquad \alpha \in (a,b)$$

$$g \in C^\infty(\mathbb{R})$$

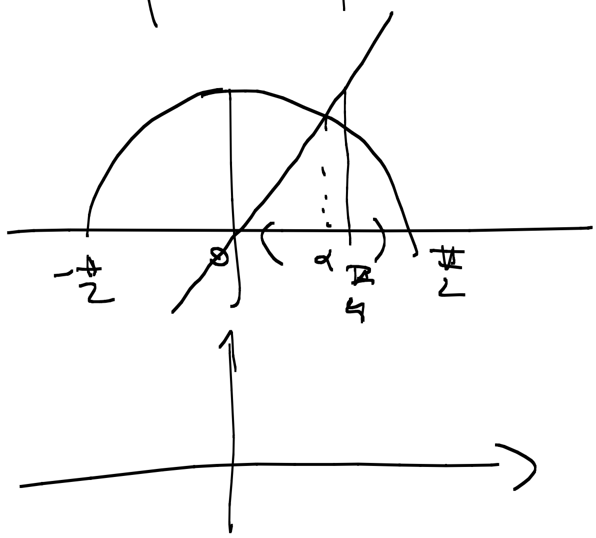
Convergenza
- $\underline{\text{Convergenza locale}} \longrightarrow |g'(\alpha)| = ?$
- Convergenza in grande in largo $\longrightarrow$ Teorema del punto fisso

$g'(x) = -\sin x \qquad |g'(x)| = |\sin x|$

$|g'(\alpha)| = |\sin \alpha| < 1 \qquad \alpha \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow$ il metodo è localmente convergente

$|g'(x)| = |\sin x| < 1$ se $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

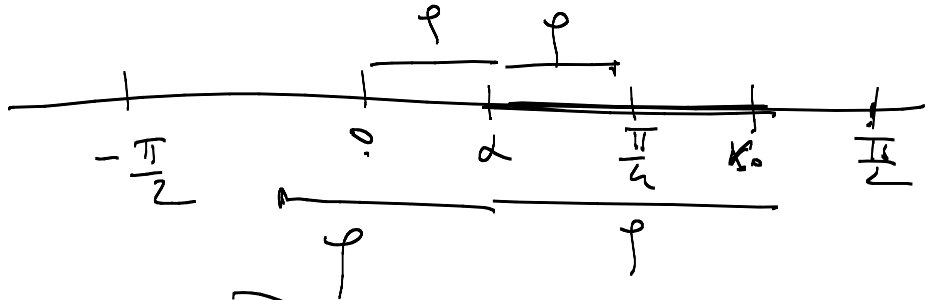


$\frac{\pi}{4} > \frac{\sqrt{2}}{2}$

$2\pi > 4\sqrt{2}$

$\pi > 2\sqrt{2}$

$\pi^2 > 8$



$x_0 = \frac{\pi}{2} \qquad x_1 = \cos\frac{\pi}{2} = 0$

$\forall x_0 \in \left[0, \frac{\pi}{2}\right] \qquad \underline{x_k \to \alpha}$

$f(x) = x^3 - 7x + 2 = 0$

$\exists! \, \alpha$ t.c. $f(\alpha) = 0 \quad \alpha \in [0, 1]$

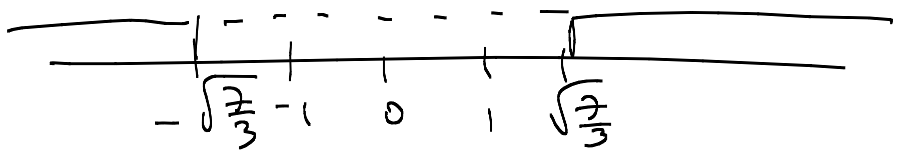
$f(0) = 2 \qquad f(1) = -4$

$\exists \, \alpha$ t.c. $f(\alpha) = 0 \quad \alpha \in [0, 1]$

Teorema esistenza degli zeri

$f'(x) = 3x^2 - 7$

$3x^2 - 7 = 0 \iff x = \pm \sqrt[2]{\frac{7}{3}}$



in $[0, 1]$ la funzione è monot. decrescente $\Rightarrow \exists! \, \alpha$

$$x_{k+1} = \frac{x_k^3 + 2}{7}$$

$f(x) = x^3 - 7x + 2$

$$x^3 - 7x + 2 = 0 \iff x^3 + 2 = 7x \iff$$

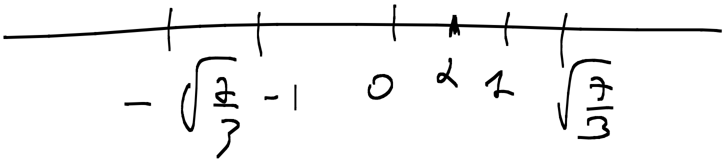
$$x = \frac{x^3 + 2}{7} \qquad x_{k+1} = g(x_k)$$

$$g(x) = \frac{x^3 + 2}{7}$$

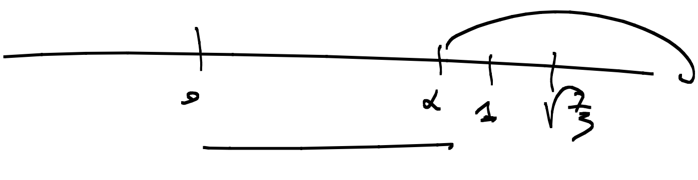
$$g \in C^1(\mathbb{R}) \quad \left(g \in C^\infty(\mathbb{R}) \right)$$

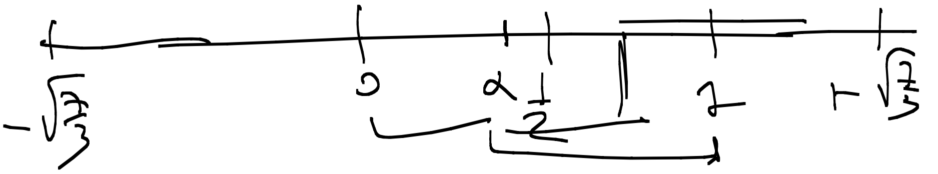
$$|g'(x)| = \left| \frac{3x^2}{7} \right| < 1$$

$$\frac{3x^2}{7} < 1 \iff 3x^2 - 7 < 0$$



$$|g'(x)| < 1 \iff -\sqrt{\frac{7}{3}} < x < \sqrt{\frac{7}{3}}$$





$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 7 \cdot \frac{1}{2} + 2$$

$$= \frac{1}{8} - \frac{7}{2} + 2 < 0$$

$\rho = 1 - \alpha$ $\qquad [\alpha - \rho, \alpha + \rho] = [2\alpha - 1, 1] \subset \left[-\sqrt{\frac{7}{3}}, \sqrt{\frac{7}{3}}\right]$

$$[0, 1] \subset [2\alpha - 1, 1] \subset \left[-\sqrt{\frac{7}{3}}, +\sqrt{\frac{7}{3}}\right]$$

$$A = \begin{bmatrix} 2 & & & -1 \\ -1 & \ddots & & \vdots \\ & \ddots & \ddots & -1 \\ & & -1 & 2 \end{bmatrix}$$

$$M = \begin{bmatrix} 2 & & \\ -1 & \ddots & \\ & \ddots & \ddots \\ & & -1 & 2 \end{bmatrix} \qquad N = \begin{bmatrix} & & 1 \\ & & \vdots \\ 0 & \cdots & 1 \\ & & 0 \end{bmatrix}$$

$$G = M^{-1} N$$

$$G = M^{-1} N = M^{-1} \begin{bmatrix} 0 & \cdots & 0 & \begin{matrix} 1 \\ 1 \\ 0 \end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \cdots & 0 & M^{-1} \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \end{bmatrix} \end{bmatrix}$$

$$M^{-1} \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \end{bmatrix} = v \iff M v = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \end{bmatrix}$$

~~inverso~~ risolvo il sistema [illegible]

$$\begin{bmatrix} 2 & & \\ -1 & \ddots & \\ & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \end{bmatrix}$$

$2 v_1 = 1 \qquad v_1 = 1/2$

$-v_1 + 2 v_2 = 1 \qquad v_2 = \dfrac{1 + \frac{1}{2}}{2} = \frac{1}{2} + \frac{1}{4}$

$-v_2 + 2 v_3 = 1 \qquad 2 v_3 = 1 + v_2 = \dfrac{1 + \frac{1}{2} + \frac{1}{4}}{2}$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

$$V_{n-1} = \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^{n-1}}$$

$$-V_{n-1} + 2V_n = 0 \qquad V_n = \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$

$$\overline{\underbrace{\begin{bmatrix} 2 & & & \\ -1 & \ddots & & \\ & \ddots & \ddots & \\ & & -1 & 2 \end{bmatrix}}_{\text{Toeplitz}} \begin{bmatrix} \frac{1}{2} & & & \\ \frac{1}{4} & \ddots & & \\ \frac{1}{8} & \ddots & \ddots & \\ \frac{1}{2^n} & \cdots \frac{1}{8} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} I \end{bmatrix}}$$

$$f(x) = x - \frac{\pi}{2} - \operatorname{arctg}(x) = 0$$

$\exists!\ \alpha$ tc $f(\alpha) = 0 \qquad \alpha \in \left[\frac{\pi}{2}, \pi\right]$

$$f'(x) = 1 - \frac{1}{1+x^2} = \frac{1+x^2-1}{1+x^2} = \frac{x^2}{1+x^2} \geq 0$$
$\forall x$

$$f\left(\frac{\pi}{2}\right) < 0 \qquad f(\pi) > 0$$

$$x_{k+1} \quad g(x_k) = \frac{\pi}{2} + \operatorname{arctg}(x_k)$$

$$g(x) = \frac{\pi}{2} + \operatorname{arctg}(x)$$

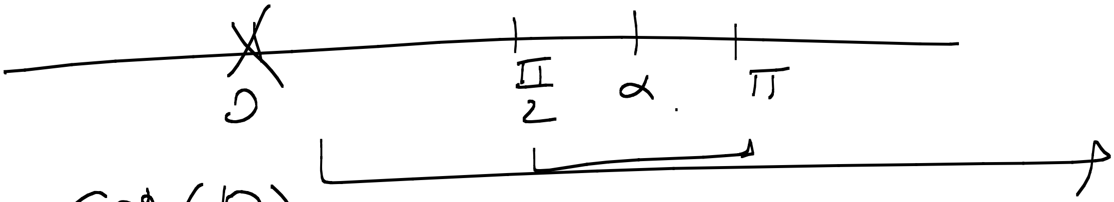
$$g(\alpha) = \frac{\pi}{2} + \operatorname{arctg}(\alpha) = \alpha \qquad \text{ok}$$

$$g'(x) = \frac{1}{1+x^2} > 0$$

$$\frac{1}{1+x^2} < 1 \iff 1 < 1+x^2$$

$$\iff 0 < x^2 \iff x \neq 0$$

$$\boxed{\begin{array}{l} g'(x) \in (0, 1] \\ g'(x) = 1 \iff x = 0 \end{array}}$$

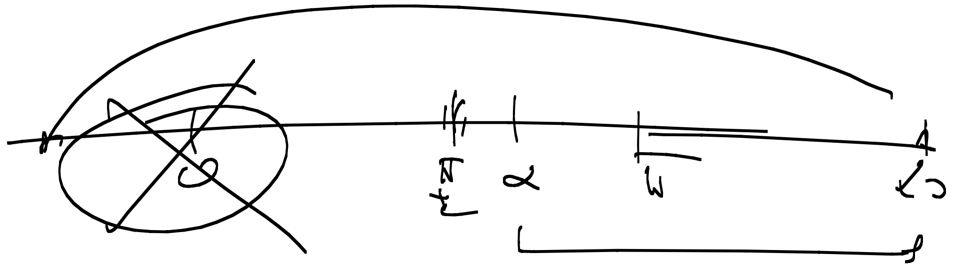


$$g \in C^\infty(\mathbb{R})$$

$$\varphi = \frac{\pi}{2} \qquad \left[\alpha - \frac{\pi}{2},\ \alpha + \frac{\pi}{2}\right]$$

• Genera succes. convergen. ad α $\forall x^{(0)}$

X



$$x_0 \geq \alpha \qquad x_1 = g(x_0)$$

Lagrange

$$x_1 - \alpha = g(x_0) - g(\alpha) = g'(\xi)\cdot(x_0 - \alpha)$$

$$\xi \in [\alpha, x_0]$$

$$x_1 - \alpha = g'(\xi)(x_0 - \alpha) = \frac{1}{1+\xi^2}(x_0 - \alpha)$$

$$\Rightarrow \quad x_1 - \alpha \geq 0 \qquad \underline{x_1 \geq \alpha}$$

$$x_1 - \cancel{\alpha} \leq x_0 - \cancel{\alpha} \qquad \underline{x_1 \leq x_0}$$

$x_0 \geq \alpha$ $\quad \{x_k\}$ sta a destra di α
monotona non crescente

$$x_k \to \ell \quad \text{finito}$$

$$x_{k+1} = g(x_k) \qquad \underline{\ell = g(\ell)}$$

$$\Rightarrow \ \underline{\ell = \alpha}$$

$$A = \begin{bmatrix} \alpha+1 & -1 & \cdots & -1 \\ -1 & \ddots & & \vdots \\ \vdots & & \ddots & -1 \\ -1 & \cdots & -1 & \alpha+1 \end{bmatrix}$$

$$\underline{\alpha+1 \neq 0 \qquad \alpha \neq -1}$$

$$J = \begin{bmatrix} 0 & \frac{1}{\alpha+1} & \cdots & \frac{1}{\alpha+1} \\ \frac{1}{\alpha+1} & \ddots & & \vdots \\ \vdots & & \ddots & \\ \frac{1}{\alpha+1} & \cdots & \frac{1}{\alpha+1} & \frac{1}{\alpha+1} \end{bmatrix}$$

$$K_1 = \dots = K_n = \left\{ z \in \mathbb{C} : |z - 0| \leq \frac{2}{|\alpha+1|} \right\}$$

tutti i cerchi hanno centro in 0 e raggio $\frac{2}{|\alpha+1|}$

Sufficiente: $\quad \frac{2}{|\alpha+1|} < 1 \iff 2 < |\alpha+1|$

$$|\alpha+1| > 2$$

$$\begin{cases} \alpha+1 > 2 \\ \alpha+1 \geq 0 \end{cases} \cup \begin{cases} -\alpha-1 > 2 \\ \alpha+1 < 0 \end{cases}$$

$$\boxed{\alpha > 1 \quad \cup \quad \alpha < -3}$$

$$J = \begin{bmatrix} & \\ & \end{bmatrix}$$

Somma per riga dei coefficienti

$$\frac{2}{\alpha+1}$$

$$J \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \frac{2}{\alpha+1} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \rightarrow \frac{2}{\alpha+1} \text{ è autovalore di } J$$

$$\left| \frac{2}{\alpha+1} \right| < 1 \quad \text{è anche necessaria.}$$

$$A = I + \alpha e e^T$$

$$M = I \qquad N = -\alpha e e^T$$

$$M x^{(k+1)} = N x^{(k)} + b$$

$$x^{(k+1)} = -\alpha e \left(e^T x^{(k)} \right) + b$$

$s = e^T x^{(k)}$; $s = -\alpha \cdot s$;

for $j = 1:n$; $x_j^{(k+1)} = b_j - s$; end.