RICEVIMENTO 22/05

$f(x) = x^2 + \sqrt{x} - 1 = 0$

$f \in C^0([0, +\infty))$

$f \in C^2((0, +\infty))$

$\lim_{x \to +\infty} f(x) = +\infty \qquad \lim_{x \to 0^+} f(x) = f(0) = -1$

$f'(x) = 2x + \frac{1}{2} x^{\frac{1}{2}-1} = 2x + \frac{1}{2\sqrt{x}} > 0 \ \forall x > 0$

$\Rightarrow \exists! \ \alpha \text{ t.c. } f(\alpha) = 0$

$f(\frac{1}{2}) < 0 \qquad f(1) > 0$

$f(1) = 1 + 1 - 1 = 1 > 0$

$f(\frac{1}{2}) = \frac{1}{4} + \frac{1}{\sqrt{2}} - 1 = \frac{\sqrt{2} + 4 - 4\sqrt{2}}{4\sqrt{2}} < 0$

$f'(x) = 2x + \frac{1}{2} x^{-\frac{1}{2}}$

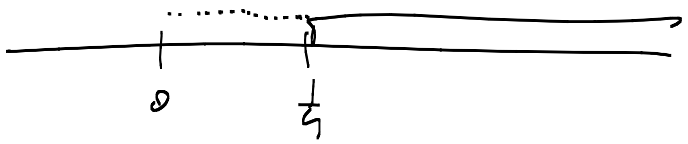
$f''(x) = 2 - \frac{1}{4} x^{-\frac{3}{2}} = 2 - \frac{1}{4} \frac{1}{x\sqrt{x}}$

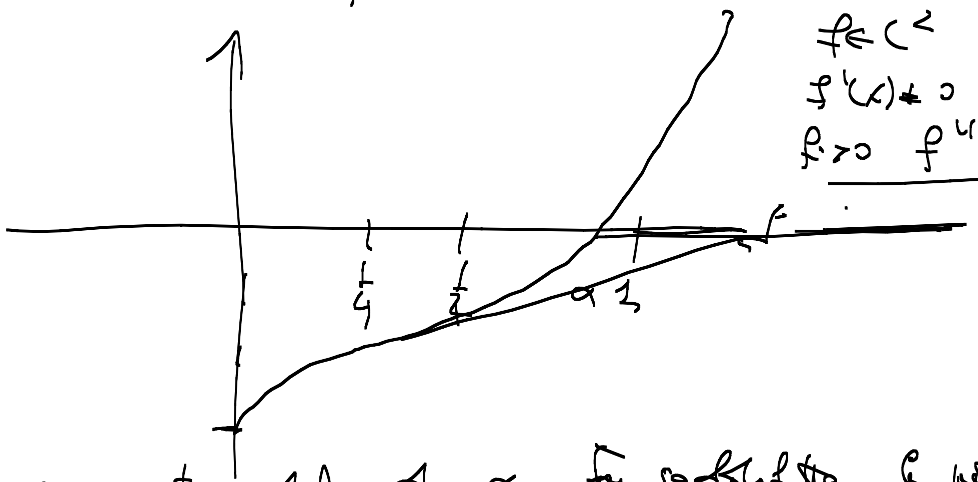
$$f''(x) \geq 0 \Rightarrow 2 - \frac{1}{4}\frac{1}{x\sqrt{x}} \geq 0$$

$$\Leftrightarrow 2 \geq \frac{1}{4}\frac{1}{x\sqrt{x}} \Leftrightarrow 8x\sqrt{x} \geq 1$$

$$\Leftrightarrow x\sqrt{x} \geq \frac{1}{8} \Leftrightarrow x^3 \geq \frac{1}{64} \Leftrightarrow$$

$$x \geq \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$$





$f \in C^2$

$f'(x) \neq 0$

$f>0 \quad f''>0 \Rightarrow f \cdot f'' > 0$

In ogni intorno [illegible] di α [illegible]

$\Rightarrow \forall x_0 \geq \alpha \quad x_k \to \alpha$

$$f(x) = x + 1 - \sqrt{1 - e^x} = 0$$

$$1 - e^x \geq 0 \Leftrightarrow 1 \geq e^x \Leftrightarrow e^x \leq 1$$

$$\Leftrightarrow x \leq 0$$

$f \in C^\infty(\mathbb{R}^-)$ $\qquad \mathbb{R}^- = \{x \in \mathbb{R} : x < 0\}$

$\lim_{x \to 0^-} f(x) = f(0) = 1$

$\lim_{x \to -\infty} f(x) = -\infty$

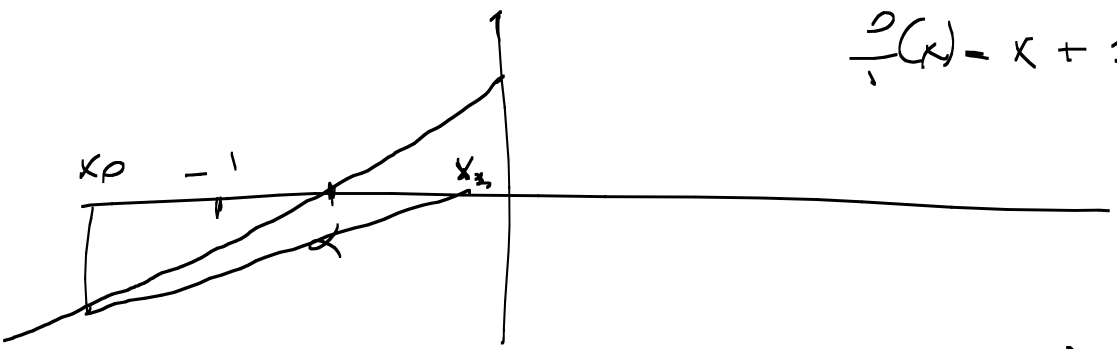
$f'(x) = 1 - \frac{1}{2} f(x)^{\frac{1}{2}-1} \cdot f'(x)$

$= 1 - \frac{f'(x)}{2\sqrt{f(x)}} = 1 + \frac{e^x}{2\sqrt{1-e^x}} > 0$

$\forall x < 0$

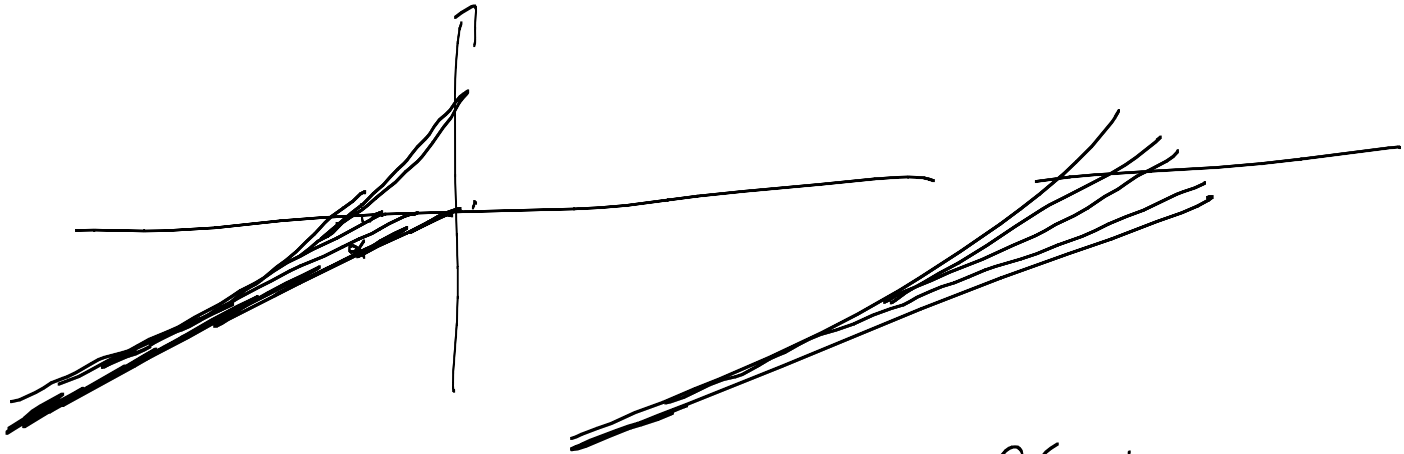
$f''(x) = \frac{1}{2} \frac{e^x \cdot \sqrt{1-e^x} + \frac{1}{2\sqrt{1-e^x}} e^x e^x}{(1-e^x)} > 0 \ \forall x < 0$

$f(x) = x + 1 - \sqrt{1-e^x}$

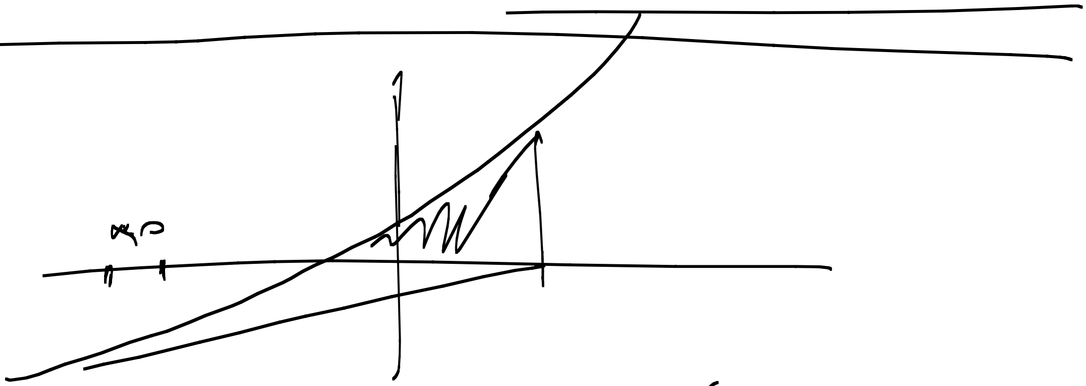


$\exists!\ \alpha$ t.c. $f(\alpha) = 0$ $\qquad \alpha \in [-1, 0)$

$\forall x_0$ con $\alpha \leq x_0 < 0$ $\qquad x_k \to \alpha$



$$x_0 < \alpha \Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} < 0$$



$$x_1 - \alpha = g(x_0) - g(\alpha) = g'(\xi)\,(x_0 - \alpha)$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\sqrt{x^3} - 3x + 1 = 0$$

$$x^3 \geq 0 \iff x \geq 0$$

$\sqrt{x^3} - 3x + 1 = 0 \iff \sqrt{x^3} = 3x - 1$

$\iff x^3 = (3x-1)^2$ e $3x - 1 \geq 0$

e $x \geq \frac{1}{3}$

$$A = \begin{bmatrix} 1 & -1 & \cdots & -1 \\ & \ddots & & \vdots \\ & & \ddots & -1 \\ 1 & & & 1 \end{bmatrix}$$

$$A \to A^{(1)} \to \begin{bmatrix} 1 & -1 & \cdots & \cdots & -1 \\ & 1 & -1 & \cdots & -1 \\ & & \ddots & & \vdots \\ 0 & 1 & \cdots & 1 & -1 \\ & & & & 2 \end{bmatrix} \qquad E_1 = \begin{bmatrix} 1 & & \\ & \ddots & \\ -1 & & 1 \end{bmatrix}$$

$$A^{(2)} \to \begin{bmatrix} 1 & -1 & \cdots & \cdots & -1 \\ & 1 & -1 & \cdots & -1 \\ & & \ddots & 1 & -1 \\ 0 & 0 & 2 & \cdots 2 & 3 \end{bmatrix} \qquad E_2 = \begin{bmatrix} 1 & & \\ & 1 & \\ & \ddots & \\ & -1 & 1 \end{bmatrix}$$

$$A^{(3)} \to \begin{bmatrix} 1 & -1 & \cdots & \cdots & -1 \\ & 1 & -1 & \cdots & -1 \\ & & \ddots & 1 & -1 \\ 0 & 0 & 0 & 4 \cdots 4 & 5 \end{bmatrix} \qquad E_3 = \begin{bmatrix} 1 & & \\ & 1 & \\ & \ddots & \\ & -2 & 1 \end{bmatrix}$$

Term. traccia dell'ultima riga di $A^{(k)} \to$ moltiplicatori

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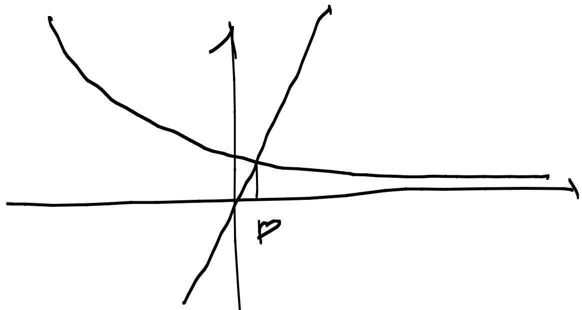
$f(x) = 4x^3 - e^{-x}$

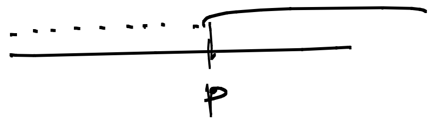
$f \in C^{\infty}(\mathbb{R})$

$\lim_{x \to +\infty} f(x) = +\infty \qquad \lim_{x \to -\infty} = -\infty$

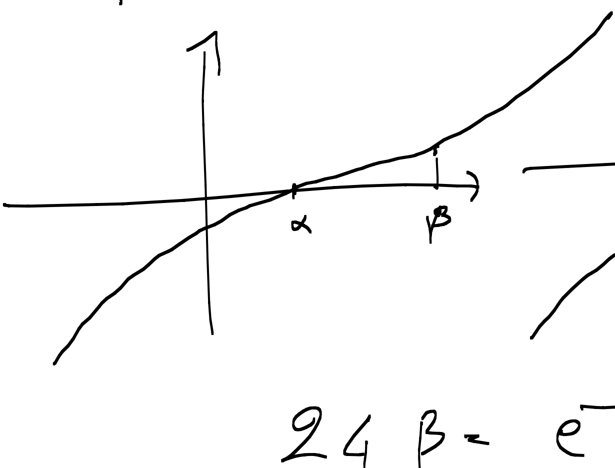
$f'(x) = 12x^2 + e^{-x} > 0 \quad \forall x \in \mathbb{R}$

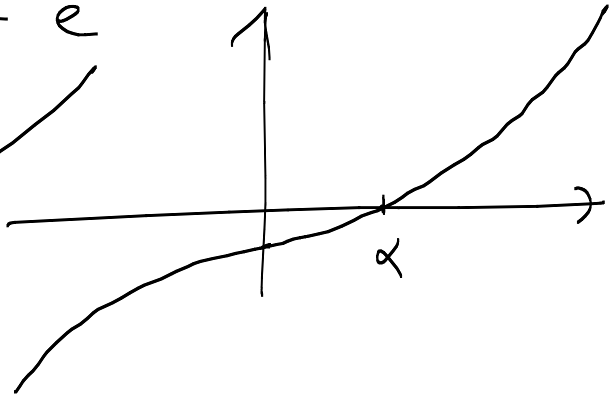
$f''(x) = 24x - e^{-x} \geq 0 \Rightarrow 24x \geq e^{-x}$





$f(x) = 4x^3 - e^{-x}$



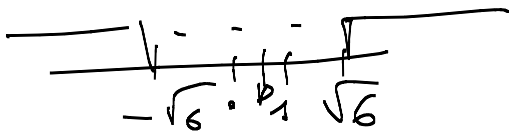


$24\beta = e^{-\beta}$

$f(\beta) = 4\beta^3 - e^{-\beta} = 4\beta^3 - 24\beta = 4\beta(\beta^2 - 6)$

$\beta^2 - 6 \geq 0$ ~~$\Leftrightarrow \beta \geq \pm\sqrt{6}$~~

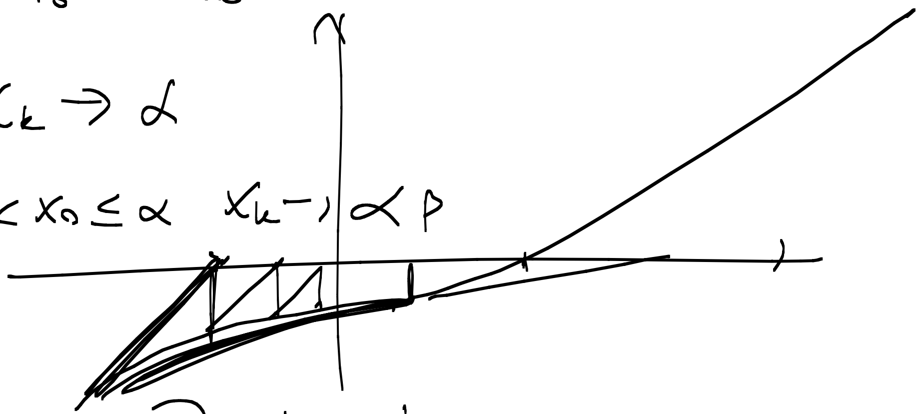
~~$\lim_{x\to+\infty} f(x) = f(+\infty) = +\infty$~~



$f(\beta) < 0$

$\forall x_0 \geq \alpha \quad x_k \to \alpha$

$\forall x_0$ con $\beta < x_0 \leq \alpha \quad x_k \to \alpha$



$x_0 < \beta \quad \exists k$ t.c. $x_k > \beta \to x_k \to \alpha$

Per assurdo: ho $x_0 < \beta$ t.c. $x_k \leq \beta \ \forall k \in \mathbb{N}$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \qquad x_{k+1} \geq x_k$$

Successioni monotone non decrescenti ho limite finito

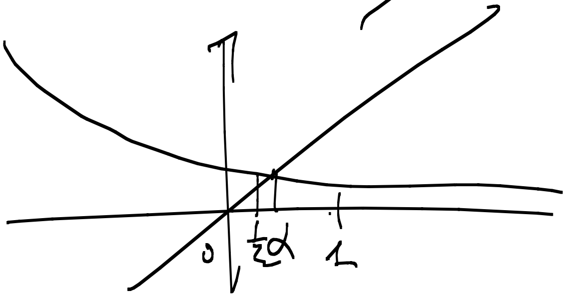
Chiamo l il lim.

$$l = l - \frac{f(l)}{f'(l)} \iff f(l) = 0$$

$$x = \frac{e^{-x} + \omega x}{1+\omega} \qquad (\omega \neq -1)$$

$$x = \frac{e^{-x} + \omega x}{1+\omega} \iff (1+\omega)x = e^{-x} + \omega x$$

$$x + \omega x = e^{-x} + \omega x \iff e^{-x} = x$$



$$e^{-\frac{1}{2}} = \frac{1}{e^{\frac{1}{2}}} = \frac{1}{\sqrt{e}} > \frac{1}{2}$$

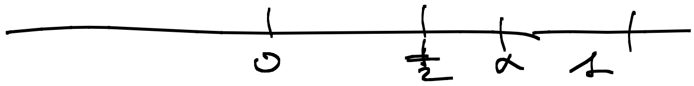
$$x = e^{-x} \qquad x_{k+1} = e^{-x_k} \quad k \geq 0$$

$$|g'(x)| = |-e^{-x}| = e^{-x}$$

$$g(x) = +e^{-x} \qquad |g'(x)| < 1 \iff e^{-x} < 1$$

$$\iff x > 0$$

$\alpha > 0 \Rightarrow |g'(\alpha)| < 1 \Rightarrow$ Quindi è localmente convergente



$\rho = \frac{1}{2}$ $\left[\alpha - \frac{1}{2}, \alpha + \frac{1}{2}\right]$ ed per la conv.

$$\left[\frac{1}{2}, 1\right] \subseteq \left[\alpha - \frac{1}{2}, \alpha + \frac{1}{2}\right]$$

$$f(x) = \sqrt{x^3} - 3x + 1 = 0$$

$x \geq 0 \qquad f \in C^{\infty}(\mathbb{R}^+)$

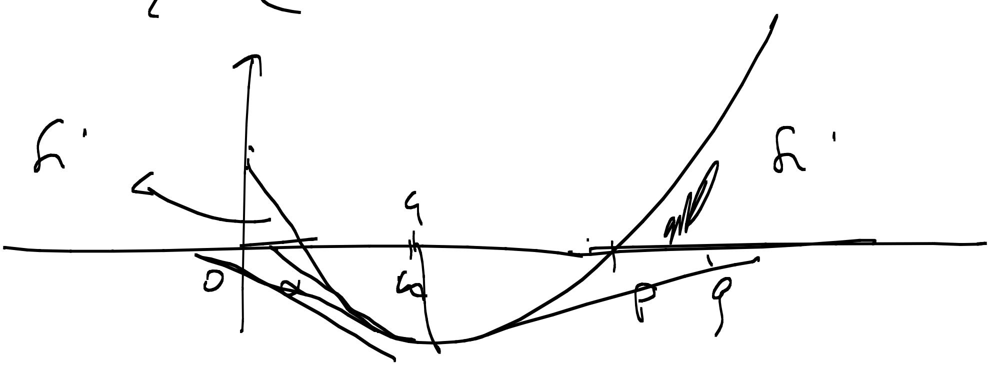
$$\lim_{x \to 0^+} f(x) = 1 \qquad \lim_{x \to +\infty} f(x) = +\infty$$

$$f'(x) = \frac{3}{2} x^{\frac{1}{2}} - 3 = \frac{3}{2}\sqrt{x} - 3$$

$$\frac{3}{2}\left(\sqrt{x} - 2\right) \geq 0 \iff \sqrt{x} - 2 \geq 0 \iff \sqrt{x} \geq 2$$

$$\iff x \geq 4$$

$$f''(x) = \frac{3}{2} \cdot \frac{1}{2} x^{-\frac{1}{2}} > 0 \quad \forall x \in \mathbb{R}^+$$



$\alpha \in [0, 1)$ $\quad\quad$ $\beta \in [4, 9)$

$f(x) = \sqrt{x^3} - 3x + 2 = x\sqrt{x} - 3x + 2$

$x_0 > 4 \quad\quad x_k \to \beta$

$0 < x_0 \leq \alpha \quad\quad x_k \to \alpha$

$\alpha < x_0 < 4 \longrightarrow x_k \to \alpha$ se $x_1 > 0$

$$x_0 - \frac{f(x_0)}{f'(x_0)} > 0$$