

Eulero esplicito: $f: [a,b] \times \mathbb{R}^m \rightarrow \mathbb{R}^m$

$$y_{n+1} = y_n + h f(t_n, y_n)$$

$$t = t_0 + \sigma h$$

$$Y = [y_0 | y_1 | y_2 | \dots | y_N]$$

$$y(t_0) = y_0$$

$$e_n = y(t_n) - y_n$$

$$E = \max_{n=1, \dots, N} \|e_n\| = \mathcal{O}(h^p)$$

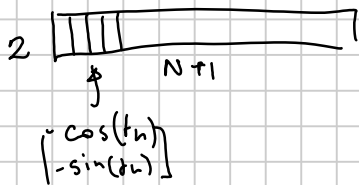
$p=1$ per E.E.

```

for n = 1, 2, ... N
    e(n) = [cos(t(n+1)); -sin(t(n+1))] - Y(:, n+1);
end

```

$$\max_{n=0, \dots, N} \max_{i=1, 2} |[\cos(t) ; -\sin(t)] - Y} = \max_{n=0, \dots, N} \|e_n\|_{\infty} = E$$



$$E = \max \left(\max \left(\text{abs} \left([\cos(t) ; -\sin(t)] - Y \right) \right) \right)$$

Eulero implicito

$$y_{n+1} = y_n + h f(t_{n+1}, y_{n+1})$$

\uparrow

\uparrow

$$y' = Ay$$

$$f(t, y) = Ay$$

$$\text{E.T. } I \cdot y_{n+1} = y_n + h f(t_{n+1}, y_{n+1}) = y_n + h A y_{n+1}$$

$1 - hA$

$$(I - hA) y_{n+1} = y_n \Rightarrow y_{n+1} = (I - hA)^{-1} y_n$$

$$y_{n+1} = (\text{eye}(m) - h * A) \setminus y_n$$

$y(:, n+1)$ $y(:, n)$

METODO DEI TRAPEZI:

$$y_{n+1} = y_n + h \left(\frac{1}{2} f(t_n, y_n) + \frac{1}{2} f(t_{n+1}, y_{n+1}) \right)$$
$$= y_n + \frac{1}{2} h A y_n + \frac{1}{2} h A y_{n+1}$$

$$\left(I - \frac{h}{2} A \right) y_{n+1} = y_n + \frac{1}{2} h A y_n = \left(I + \frac{h}{2} A \right) y_n$$

$$\Rightarrow y_{n+1} = \left(I - \frac{h}{2} A \right)^{-1} \left(I + \frac{h}{2} A \right) y_n$$