RICEVIMENTO 23/04

$$x = Px + q$$

$$\begin{cases} x^{(0)} \in \mathbb{R}^n \\ x^{(k+1)} = P x^{(k)} + q \end{cases}$$

$$\lim_{k \to +\infty} x^{(k+1)} = v \Rightarrow v = Pv + q$$

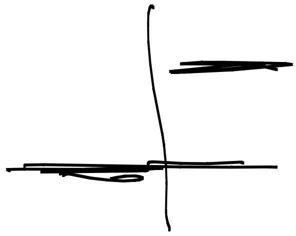
$$\Rightarrow Av = b$$

$$f : \mathbb{R} \to \mathbb{R}$$

$$x_{k+1} = f(x_k)$$

$$x_k \to l \qquad l = f(l)$$

$$f(x) = \begin{cases} 0 & \text{se } x \leq 0 \\ 1 & \text{se } x > 0 \end{cases}$$



$$x_k = \frac{1}{k} \qquad x_k \to 0$$

$$f(x_k) = 1 \qquad f(x_k) \to 1$$

$$x^{(k+1)} = P x^{(k)} + q$$

$$v = Pv + q$$

$$x^{(k+1)} = P x^{(k)} + q$$

$$v = Pv + q$$

$$A = \begin{bmatrix} 2 & -1 & & \\ & \ddots & \ddots & \\ & & \ddots & -1 \\ -2 & & & 2 \end{bmatrix}$$

$A^{(1)}$ dopo 1 passo di elim. gaussiana

$A = A^{(1)}$ per matrici.

$$A \to A^{(1)} = \begin{bmatrix} 2 & -1 & & \\ & \ddots & \ddots & \\ & & \ddots & -1 \\ 0 & -1 & & 2 \end{bmatrix}$$

$$A^{(1)} = \left[\begin{array}{c|ccc} 2 & -1 & & \\ \hline & 2 & -1 & \\ 0 & & \ddots & -1 \\ & -1 & & 2 \end{array}\right]$$

$$A = \left[\begin{array}{c|c} B & \\ \hline & C \end{array}\right]$$

triangolari superiori a blocchi

$\det A = \det B \cdot \det C$

$$\det A^{(1)} = 2 \cdot \det \begin{bmatrix} 2 & -1 & \\ & \ddots & -1 \\ -1 & & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & & \\ & \ddots & \ddots & \\ & & \ddots & -1 \\ -1 & & & 2 \end{bmatrix}$$

predom. diagonale x righe e per [illegible] invertibili

$\det A^{(1)} \neq 0 \Rightarrow \det A = \det A^{(n)} \neq 0$

$$\begin{bmatrix} 1 & & & -\alpha \\ 2\alpha & 2 & & \\ & \ddots & \ddots & \\ & & n\alpha & n \end{bmatrix}$$

$\forall \alpha \in \mathbb{R}$ A ammette unica la fattorizzazione LU

$$\begin{bmatrix} 1 & & & \\ 2\alpha & 2 & & \\ & \ddots & \ddots & \\ & & (n-1)\alpha & n-1 \end{bmatrix} = \begin{bmatrix} 1 & & & & \\ 2\alpha & 1 & & & \\ & \frac{3\alpha}{2} & 1 & & \\ & & \frac{4\alpha}{3} & \ddots & \\ 0 & & & \frac{(n-1)\alpha}{n-2} & 1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 2 & & \\ & & \ddots & \\ & & & n-1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} a & 0 \\ c & b \end{bmatrix}}_{} = \underbrace{\begin{bmatrix} a & \\ c & b \end{bmatrix} \cdot \begin{bmatrix} a^{-1} & 0 \\ 0 & b^{-1} \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{U}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ \alpha & 2 & 0 \\ 0 & \alpha & 3 \end{pmatrix}$$

[illegible] α [illegible]

$\forall \alpha \in \mathbb{R}$

$|\alpha| < 2$

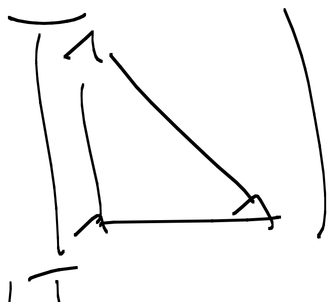
$|\alpha| < 3 \Rightarrow |\alpha| < 2$

$\Leftrightarrow -2 < \alpha < 2$

$$A = \begin{pmatrix} 1 & \cdots & \cdots & 1 \\ 1 & 2 & \cdots & 2 \\ \vdots & 3 & 3 \cdots & 3 \\ 1 & \vdots & \vdots & \\ & & 3 & \end{pmatrix}$$

$a_{ij} = \min(i;j)$

$A = LU$

$L =$ 

$U = L^T$

$$K_2(A) = \|A\|_2 \, \|A^{-1}\|_2$$

$$\|A\|_2 = \underbrace{1 + 2 + 3 + \dots + n}\; = \frac{(n+1)\,n}{2}$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

$$\underline{\|A^{-1}\|_2} \qquad A = LU$$

$$A^{-1} = (LU)^{-1} = U^{-1} \cdot L^{-1}$$

$$(LU) \cdot (LU)^{-1} = I$$

$$L \cdot U \cdot (LU)^{-1} = I$$

$$(LU)^{-1} = \underline{U^{-1} L^{-1}}$$

$$\|A^{-1}\|_1 = \|U^{-1} \cdot L^{-1}\|_1 \le \|U^{-1}\|_1 \|L^{-1}\|_1$$

$$L = \begin{pmatrix} 1 & & \\ \vdots & \ddots & \\ 1 & \cdots & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & & & \\ -1 & \ddots & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{pmatrix} = L^{-1}$$

$$\begin{pmatrix} 1 & & \\ \vdots & \ddots & \\ 1 & \cdots & 1 \end{pmatrix} \begin{pmatrix} 1 & & & \\ -1 & \ddots & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix}$$

$$U^{-1} = \begin{pmatrix} 1 & -1 & & \\ & \ddots & \ddots & \\ & & \ddots & -1 \\ & & & 1 \end{pmatrix} \qquad \|A^{-1}\|_1 \le 2 \cdot 2 = 4$$

$$\|A\|_1 \|A^{-1}\|_1 \le \frac{n(n+1)}{2} \cdot 4$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & & & \\ 1 & 1 & & \\ 1 & 1 & 1 & \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ & 1 & 1 & 1 \\ & & 1 & 1 \\ & & & 1 \end{pmatrix}$$

$$A = \begin{bmatrix} * & & \\ \vdots & \ddots & \\ * & \cdots & * \end{bmatrix} \begin{bmatrix} * & \cdots & * \\ & \ddots & \vdots \\ & & * \end{bmatrix}$$

$$A^{(1)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$E_1 = \begin{pmatrix} 1 & & & \\ -1 & 1 & & \\ -1 & & 1 & \\ -1 & & & 1 \end{pmatrix}$$

$$E_L = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & -1 & \ddots & \\ & -1 & & 1 \end{pmatrix}$$

$$L = \begin{bmatrix} 1 & & \\ \vdots & \ddots & \\ 1 & \cdots & 1 \end{bmatrix}$$