

Esercizio 28/104

Jacobi e Gauss-Seidel x nota a

procc.

$$A_c \begin{bmatrix} x_1 & v_1 & \dots & v_{n-1} \\ z_1 & & & 0 \\ \vdots & & & \\ z_{m-1} & 0 & & x_m \end{bmatrix} b$$

Approbato $x_i \neq 0 \quad i=1, \dots, m$

Condizione di arresto $\frac{\|y^{(k+1)} - y^{(k)}\|_\infty}{\|y^{(k+1)}\|_\infty} \leq \text{tol.}$

$k > 0$ iterazioni

$$M_c = \begin{bmatrix} x_1 & & & \\ & \ddots & & \\ & & & x_m \end{bmatrix}$$

$$N_c = \begin{bmatrix} 0 & -v_1 & \dots & -v_{n-1} \\ -z_1 & & & \\ \vdots & & & 0 \\ -z_{m-1} & & & \end{bmatrix}$$

Condições de erro de perturbação

$$Ax = b \quad \text{calcula } \hat{x}$$

$$r = A\hat{x} - b$$

$$r = A\hat{x} - b = A\hat{x} - Ax = A(\hat{x} - x)$$

$$\hat{x} - x = A^{-1}r$$

$$\|\hat{x} - x\| = \|A^{-1}r\| \leq \|A^{-1}\| \|r\|$$

$$\frac{\|\hat{x} - x\|}{\|x\|} \leq \frac{\|A^{-1}\| \|r\|}{\frac{\|b\|}{\|A\|}}$$

$$\|Ax\| = \|b\|$$

$$\|b\| \leq \|A\| \|x\|$$

$$\|x\| \geq \frac{\|b\|}{\|A\|}$$

$$\frac{\|x - x\|}{\|x\|} = \frac{\|A^{-1}b - x\|}{\|b\|}$$

Can you do more

$$\frac{\|A^{-1}b - b\|}{\|b\|} \text{ stop}$$