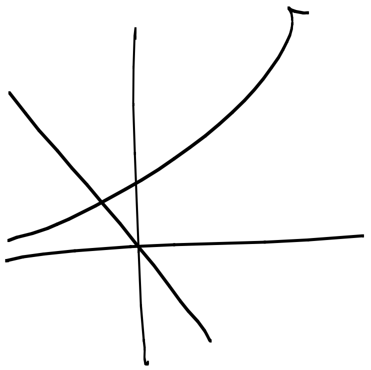
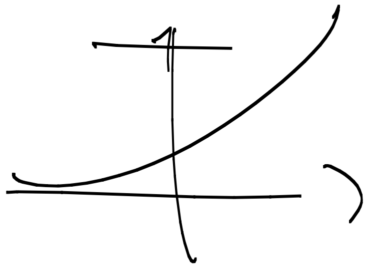
$f(x) = e^x - kx = 0 \qquad k \in \mathbb{R}$

$$e^x = kx$$

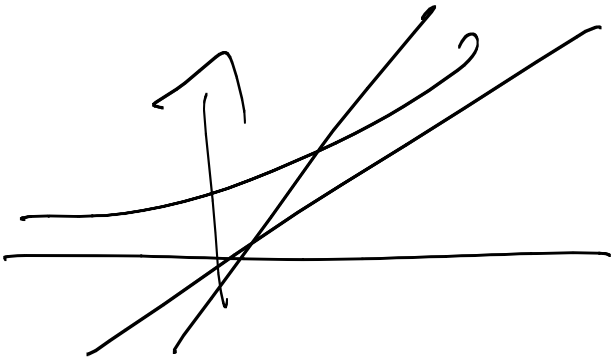


$k < 0 \quad \exists !$ soluzione.



$k = 0$

$e^x = 0$ non ha soluzioni



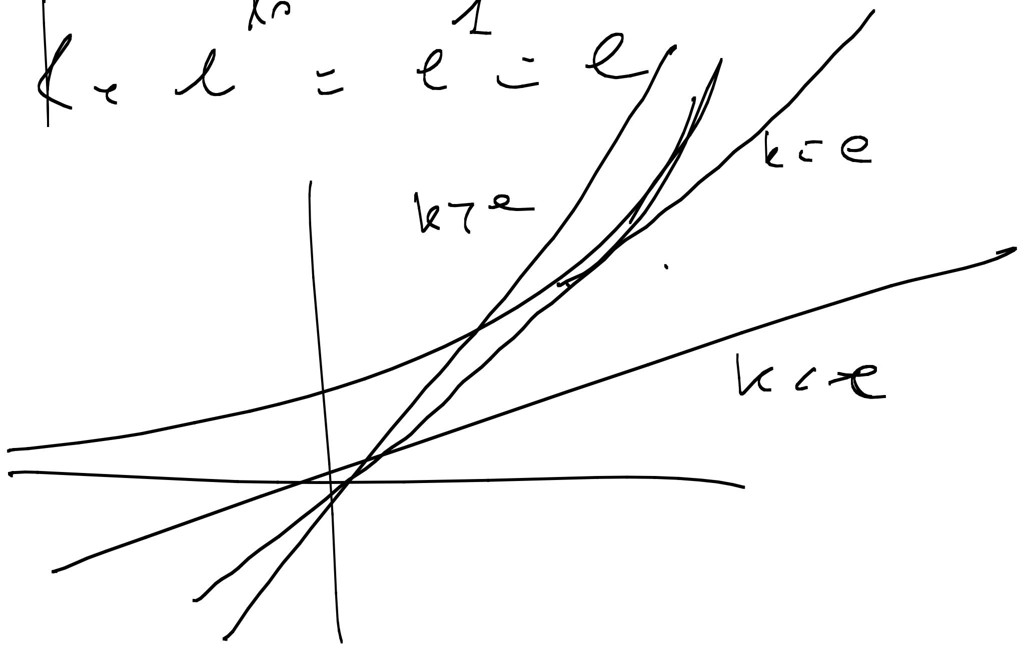
$k > 0$

$y = kx$ tangente al grafico della funzione $y = e^x$

$$\begin{cases} k x_0 = e^{x_0} \\ k = e^{x_0} \end{cases}$$

$e^{x_0} \cdot x_0 = e^{x_0} \Rightarrow x_0 = 1$

$k = e^{x_0} = e^1 = e$



$0<k<e$ non ci sono sol.

$k=e$ 1 soluzione

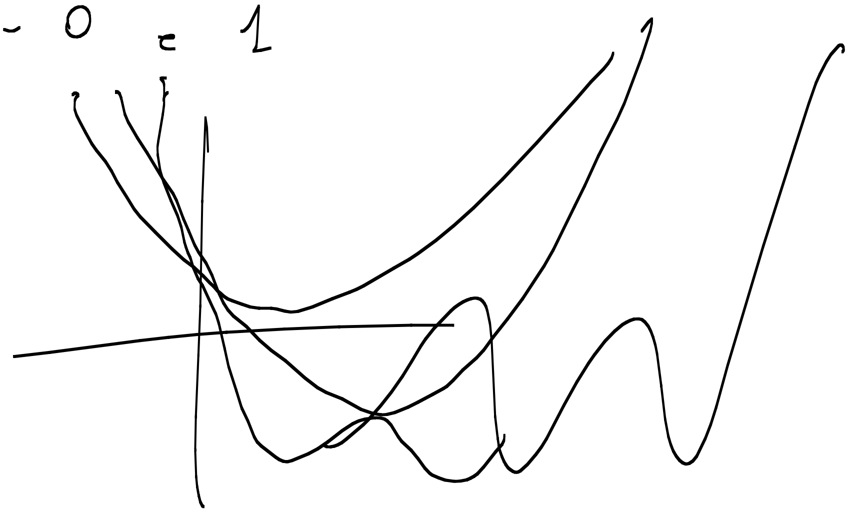
$k>e$ 2 sol. distinte

$f(x) = e^x - kx$ $\quad$ $\underline{k>0}$

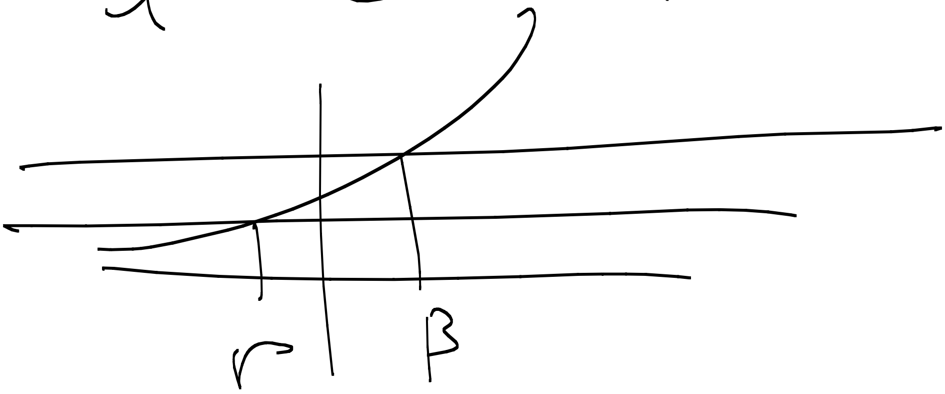
$f \in C^\infty(\mathbb{R})$ $\quad$ $\lim_{x \to -\infty} f(x) = +\infty$

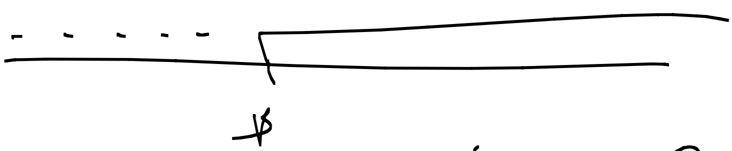
$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} e^x \cdot \left(1 - \frac{kx}{e^x}\right) = +\infty$$

$$f(0) = e^0 - 0 = 1$$



$$f'(x) = e^x - k \geq 0 \iff e^x \geq k$$



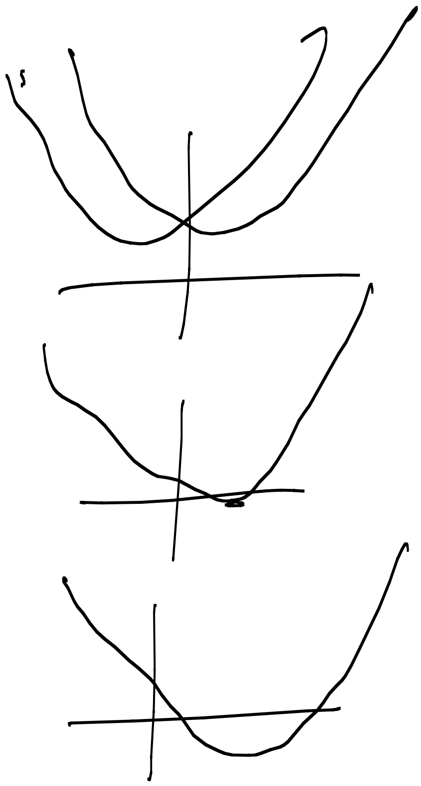


$$e^{\beta} = k \qquad \beta = \ln k$$

$$f(\ln k) = e^{\ln k} - k \ln k$$

$$= k - k\ln k = k \cdot (1 - \ln k)$$

$$1 - \ln k = 0 \iff k = e$$



$0 < k < e \quad f(\ln k) > 0$

$k = e \quad f(\ln k) = 0$

$k > e \quad f(\ln k) < 0$

$$e^x + \log x$$

$C^\infty(\mathbb{R}^{+}) \quad \lim_{x \to +\infty} f(x) = +\infty \quad \lim_{x \to 0^+} f(x) = -\infty$

$f'(x) = e^x + \frac{1}{x} \neq 0 \quad \forall x \in \mathbb{R}^+$

$f''(x) = e^x - \frac{1}{x^2}$