Esercitazione 26/05

$$f(x) = \log(e^x+1) + x - 1 = 0$$

$e^x+1 > 0 \ \forall x \in \mathbb{R} \Rightarrow f \in C^\infty(\mathbb{R})$

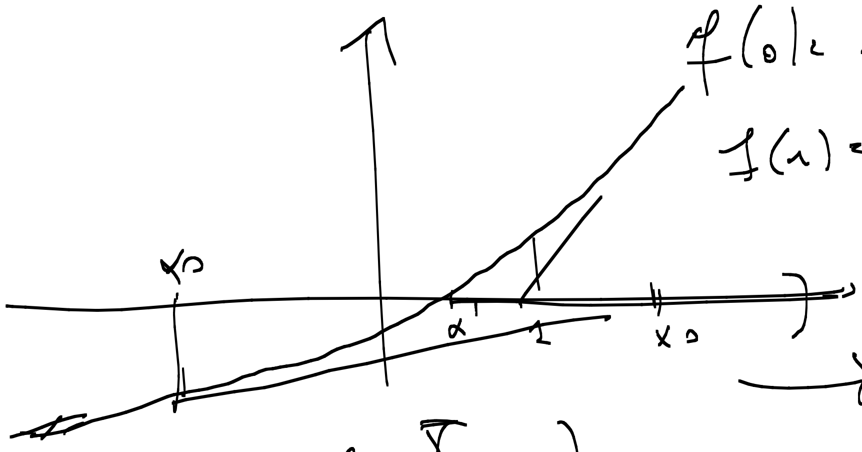
$\lim_{x\to+\infty} f(x) = +\infty \qquad \lim_{x\to-\infty} f(x) = -\infty$

$$f'(x) = \frac{1}{e^x+1} e^x + 1 = \frac{e^x}{e^x+1} + 1 > 0 \quad \forall x \in \mathbb{R}$$

$$f''(x) = \frac{e^x(e^x+1) - e^x \cdot e^x}{(e^x+1)^2} = \frac{e^x}{(e^x+1)^2} > 0 \quad \forall x \in \mathbb{R}$$

$f(0) = \log 2 - 1 < 0$

$f(1) = \log(e+1) > 0$



$\to$ ok $f'(x) > 0$, $f\, f'' > 0$

$\alpha \in [0,1]$

$f \in C^\infty(\mathbb{R})$ $f(\alpha) = 0$ $f'(\alpha) \neq 0 \Rightarrow$ il metodo delle tangenti

è localmente convergente ad α e la convergenza è localmente almeno quadratica
($f''(\alpha) \neq 0$ esattamente quadratica)

$f \in C^{\infty}(\mathbb{R})$

$\forall x_0 \geq \alpha \Rightarrow x_k \to \alpha$ (teorema di convergenza in grande)

$\forall x_0 \leq \alpha \quad x_1 \geq \alpha \Rightarrow x_k \to \alpha$

$$\log(e^x + 1) + x - 1 = 0 \iff$$
$$x = 1 - \log(e^x + 1)$$

$$x_{k+1} = 1 - \log(e^{x_k} + 1)$$

$$g(x) = 1 - \log(e^x + 1)$$

$$g \in C^{\infty}(\mathbb{R})$$

$$|g'(x)| = \left| -\frac{e^x}{e^x + 1} \right| = \frac{e^x}{e^x + 1}$$

$$\left(|g'(x)| < 1 \quad \forall x \in \mathbb{R}\right.$$

Il metodo è convergente $\forall x_0 \in \mathbb{R}$

$$\frac{e^x}{e^x + 1} < 1 \iff e^x < e^x + 1 \quad (e^x + 1 > 0 \ \forall x \in \mathbb{R})$$

$\iff 0 < 1$ sempre. $\forall x \in \mathbb{R}$

$$|g'(\alpha)| = \frac{e^\alpha}{e^\alpha + 1} < 1 \iff e^\alpha < e^\alpha + 1$$

$\iff 0 < 1$ OK

$$A = \begin{bmatrix} 1 & 2 & \cdots & n \\ -1 & & & \\ & \ddots & & \\ & & & -(n-1) \end{bmatrix}$$

$$A \to A^{(1)} = \begin{bmatrix} 1 & 2 & \cdots & n \\ 0 & 2 & \cdots & n \\ & -2 & & \\ & & \ddots & \\ & & & -n-1 \end{bmatrix} \quad E_1 = \begin{pmatrix} 1 & & \\ 1 & \ddots & \\ & & 1 \end{pmatrix}$$

$$A^{(1)} \to A^{(2)} = \begin{bmatrix} 1 & 2 & \cdots & n \\ 0 & 2 & \cdots & n \\ & 0 & 3 \cdots & n \\ & & \ddots & \end{bmatrix} \qquad E_2 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & 1 & \ddots & \\ & & & 1 \end{pmatrix}$$

$$U_2 = \begin{pmatrix} 1 & 2 & \cdots & n \\ & 2 & \cdots & n \\ & & \ddots & \vdots \\ & & & n \end{pmatrix} \qquad L_2 = \begin{pmatrix} 1 & & & \\ -1 & \ddots & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{pmatrix}$$

$$A = \underline{L\,U}$$

$$\begin{pmatrix} 1 & & & \\ -1 & \ddots & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & \cdots & n \\ & \ddots & & \vdots \\ & & \ddots & \vdots \\ & & & n \end{pmatrix} = \begin{pmatrix} 1 & 2 & \cdots & n \\ & & & \\ & & & \end{pmatrix}.$$

$= -1 \cdot$ riga $(i-1)$ di U $+ 1$ riga (i) di U

$$= -1 \begin{bmatrix} 0 & \cdots & 0 & (i-1) & i & \cdots & n \end{bmatrix} + 1 \begin{bmatrix} 0 & \cdots & 0 & i & \cdots & n \end{bmatrix}.$$

$$= \begin{bmatrix} 0 & \cdots & 0 & -(i-1) & 0 & \cdots & 0 \end{bmatrix} = \text{riga } i \text{ di } A.$$

$$A_k = \begin{bmatrix} 1 & 2 & \cdots & & k \\ -1 & & & & \\ & \ddots & & & \\ & & \ddots & & \\ & & & -(k-1) & \end{bmatrix}$$

~~Sviluppo di Laplace rispetto alla 1a riga~~

Sviluppo secondo l'ultima colonna

$$(-1)^{k+1}\, k \underbrace{(-1)\cdot(-2)\cdots(-(k-1))}_{\neq 0}$$

$$(-1)^{k+1}(-1)^{k-1}\, 1\, 2 \cdots (k-1)\, k$$

$$= k! \neq 0$$

$$\det A = \det U = 1\, 2 \cdots n = \underline{\underline{n!}}$$

$$U = \begin{pmatrix} 1 & 2 & \cdots & n \\ & 2 & \cdots & n \\ & & \ddots & \\ & & & n \end{pmatrix} \qquad \underline{Ux = b}$$

$$U = \begin{pmatrix} u_{11} & \cdots & u_{1n} \\ & \ddots & \\ & & u_{nn} \end{pmatrix}$$

$$x(n) = b(n)/u(n,n)$$

$$x(k) = \frac{b(k) - \sum_{j=k+1}^{n} u_{kj}\, x(j)}{u(k,k)}$$

$$u(k,k)\, x(k) + \sum_{j=k+1}^{n} u(k,j)\, x(j) = \underline{b(k)}$$

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function [x] = prog(b);
n = length(b);
x = zeros(n,1);
x(n) = b(n)/n;   s = 0;
  for k = n-1:-1:1
     s = s + (k+1) * x(k+1);
     x(k) = (b(k) - s)/k;
end.
```

$$U = \begin{pmatrix} 1 & \cdots & n \\ & \ddots & \vdots \\ & & n \end{pmatrix}$$

2 oper mole [illegible]

+

2 oper. [illegible]

[illegible]

$$X_k = \frac{b(k) - b(k+1)}{k}$$

$X_n = b_n/n$ $\qquad X_{n-1} = \dfrac{b_{n-1} - n\, x(n)}{(n-1)}$

$X_{n-1} = b_{n-1} -$

$A = LU \qquad U^{-1} = \begin{bmatrix} 1 & -1 & 0 & \cdots \\ & \frac{1}{2} & -\frac{1}{2} & \\ & & \ddots & \ddots \\ & & & \frac{1}{n} \end{bmatrix} \begin{bmatrix} 1 & 2 & \cdots & n \\ & 2 & 3 & \\ & & \ddots & \\ & & & n \end{bmatrix}$

$Ux = b \quad (2) \qquad x = U^{-1} b \qquad = I$

$$x_1 - \alpha = g(x_0) - g(\alpha) \qquad g(x) = x - \frac{f(x)}{f'(x)}$$

$$= g'(\xi) \cdot (x_0 - \alpha) \qquad g'(\xi) = \frac{f(\xi) f''(\xi)}{(f'(\xi))^2}$$

$$\xi \in (x_0, \alpha) \qquad f(\xi) < 0 \quad f''(\xi) > 0$$

$$\Rightarrow g'(\xi) < 0$$

$$x_0 - \alpha < 0 \Rightarrow x_1 - \alpha > 0 \Rightarrow \underline{x_1 > \alpha}$$