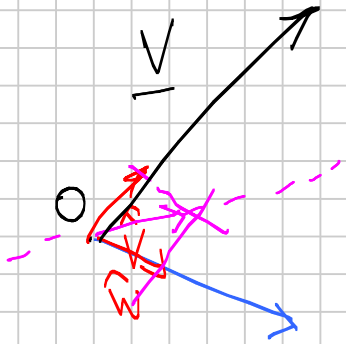
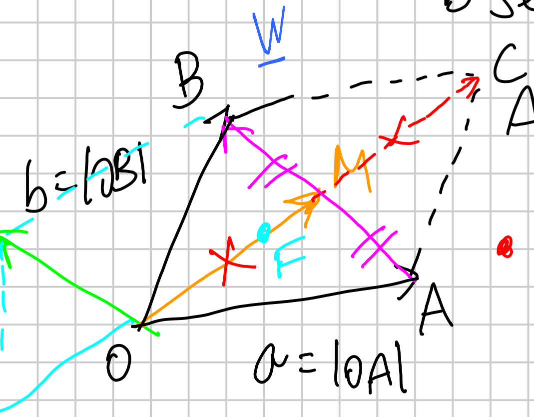
$\hat{v} = \frac{v}{|v|}$



• $\hat{v} + \hat{w}$ è la diagonale di un rombo

In questo modo si costruisce la bisettrice di un angolo



$\underline{AB} = \underline{AO} + \underline{OB} = -\underline{OA} + \underline{OB} = \underline{OB} - \underline{OA}$

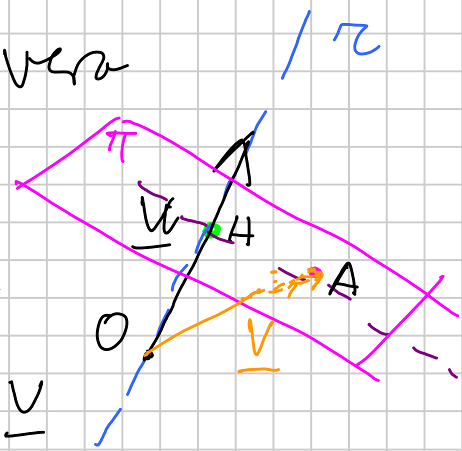
• $OM = \frac{OA+OB}{2}$

$OM = OA + AM = OA + \frac{AB}{2}$

$= OA + \frac{(OB-OA)}{2} = \frac{OA+OB}{2}$

$OC = OA + OB$

$OF = t(OA+OB)$ deve esistere t tale che questa uguaglianza sia vera



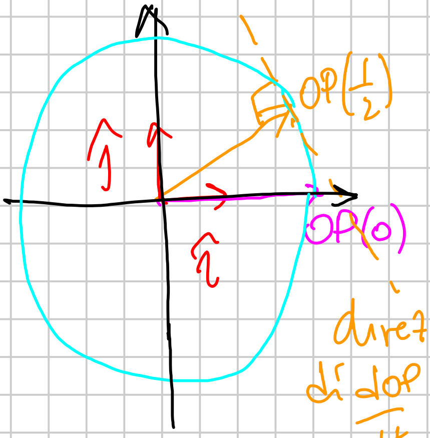
$(\underline{u} \wedge \underline{v}) \wedge \underline{w} = (\underline{u} \cdot \underline{w})\underline{v} - (\underline{v} \cdot \underline{w})\underline{u}$

$\underline{w} \wedge (\underline{u} \wedge \underline{v}) = (\underline{v} \cdot \underline{w})\underline{u} - (\underline{u} \cdot \underline{w})\underline{v}$

$\underline{u} = \underline{w}$ $\quad \underline{z} = \underline{u} \wedge (\underline{u} \wedge \underline{v}) = (\underline{u} \cdot \underline{v})\underline{u} - u^2 \underline{v}$

Esempi di vettori dipendenti da un parametro

$$\underline{OP}(t) = R\cos(t)\,\hat{\imath} + R\,\mathrm{sen}(t)\,\hat{\jmath}$$

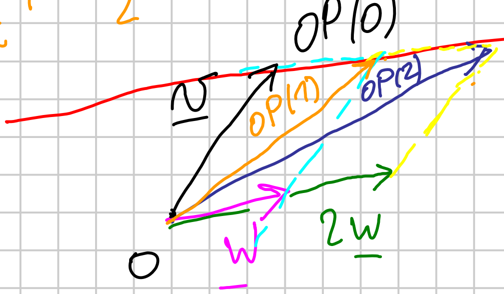


$$|OP|^2 = R^2\cos^2(t) + R^2\,\mathrm{sen}^2(t) = R^2$$

$= \text{costante}$

$$d(O,P) = R$$

$$\underline{OP}(t) = \underline{v} + t\,\underline{w}$$



tutte le punte dei vettori $OP(t)$ stanno su questa retta

$$\begin{cases} x = R\cos(t) \\ y = R\,\mathrm{sen}(t) \end{cases}$$

$z = 0$

$$\begin{cases} x = v_x + t\,w_x \\ y = v_y + t\,w_y \\ z = v_z + t\,w_z \end{cases}$$

Per la circonferenza

$$\frac{d\underline{OP}}{dt} = -R\,\mathrm{sen}\,t\,\hat{\imath} + R\cos t\,\hat{\jmath}$$

$$\underline{OP}\cdot\frac{d\underline{OP}}{dt} = R^2[\cos(t)(-\mathrm{sen}\,t) + \mathrm{sen}(t)\cdot\cos(t)]$$

$|OP| = R = \text{cost.}$

$$\frac{d\underline{OP}}{dt} \perp \underline{OP}$$

Per la retta

$$\frac{d\underline{OP}}{dt} = \underline{w}$$

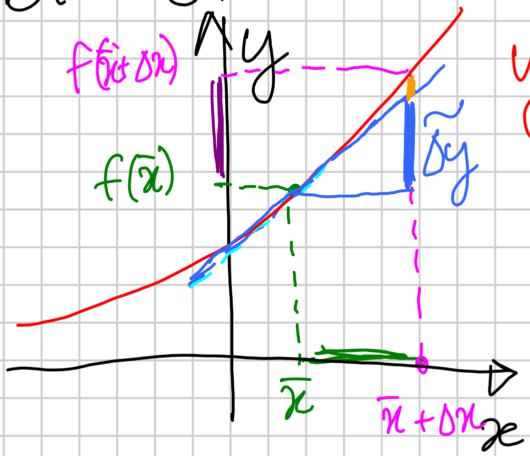
$\underline{v}(t)$ $\quad t \in [t_1, t_2]$

Cambio di variabile $\quad s$ scalare

$$\underline{v}(s(t)) \qquad \frac{d\underline{v}}{dt} = \frac{d\underline{v}}{ds}\,\frac{ds}{dt}$$

Scalare

DIFFERENZIALE DI UNA FUNZIONE



$y = f(x)$

$$\Delta y = f(\bar{x}+\Delta x) - f(\bar{x})$$

$$\tilde{\Delta y} = f'(\bar{x}) \cdot \Delta x$$

$$\Delta y - \tilde{\Delta y} = f(\bar{x}+\Delta x) - f(\bar{x}) - f'(\bar{x}) \cdot \Delta x$$

è infinitesimo di ordine superiore al primo quando $\Delta x \to 0$

se sostituisco dx (infinitesimo) al posto di Δx

$$df = f'(\bar{x})\, dx + o(dx)$$

$$f'(\bar{x}) \, \Delta x \simeq \Delta f$$

incremento della variabile dipendente

numero

incremento della variabile indipendente

$$d\underline{v} = \left(\frac{d\underline{v}}{dt}\right)_{t=\bar{t}} \cdot dt \qquad \Delta \underline{v} = \left(\frac{d\underline{v}}{dt}\right)_{t=\bar{t}} \cdot \Delta t$$

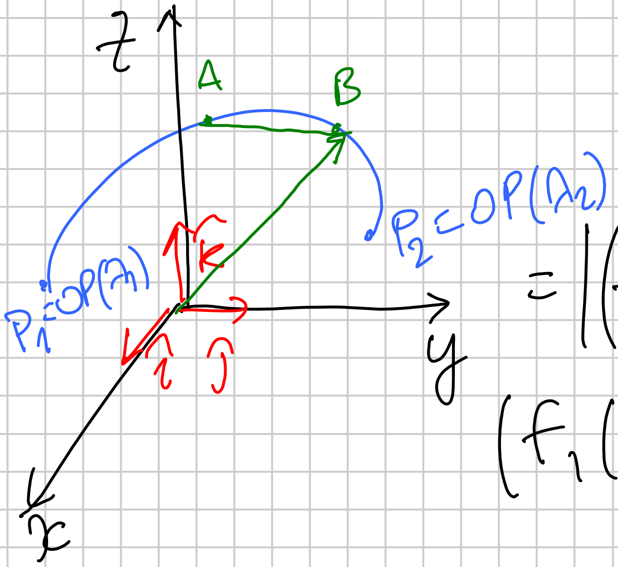
$$d\underline{v} = (dv_x \;\; dv_y \;\; dv_z) = (A \;\; B \;\; C) \cdot dt$$

op. lin da $\mathbb{R}^1$ in $\mathbb{R}^3$

$$|\Delta \underline{v}| = |(A \;\; B \;\; C) \cdot \Delta t| + o(\Delta t)$$

CALCOLO DELLA LUNGHEZZA DI UNA CURVA

$$OP = f_1(\lambda)\hat{\imath} + f_2(\lambda)\hat{\jmath} + f_3(\lambda)\hat{k}$$



$$|AB|^2 = |OB - OA|^2 =$$

$$= \left|\left(f_1(\lambda_B)\hat{\imath} + f_2(\lambda_B)\hat{\jmath} + f_3(\lambda_B)\hat{k}\right) - \left(f_1(\lambda_A)\hat{\imath} + f_2(\lambda_A)\hat{\jmath} + f_3(\lambda_A)\hat{k}\right)\right|^2$$

$$= \left|\left(f_1(\lambda_B) - f_1(\lambda_A)\right)\hat{\imath} + \left(f_2(\lambda_B) - f_2(\lambda_A)\right)\hat{\jmath} + \left(f_3(\lambda_B) - f_3(\lambda_A)\right)\hat{k}\right|^2$$

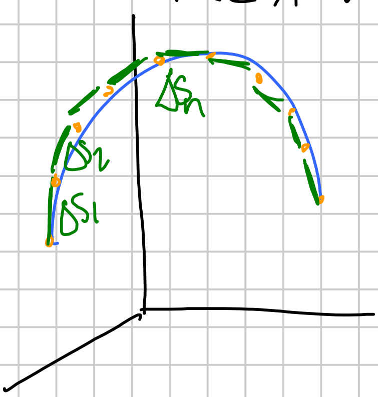
$$= \left[\left(f_1(\lambda_B) - f_1(\lambda_A)\right)^2 + \left(f_2(\lambda_B) - f_2(\lambda_A)\right)^2 + \left(f_3(\lambda_B) - f_3(\lambda_A)\right)^2\right] =$$

$\Delta\lambda = \lambda_B - \lambda_A$

$$= \left(\left.\frac{df_1}{d\lambda}\right|_{\lambda_A} \cdot \Delta\lambda\right)^2 + \left(\left.\frac{df_2}{d\lambda}\right|_{\lambda_A} \cdot \Delta\lambda\right)^2 + \left(\left.\frac{df_3}{d\lambda}\right|_{\lambda_A} \cdot \Delta\lambda\right)^2 =$$

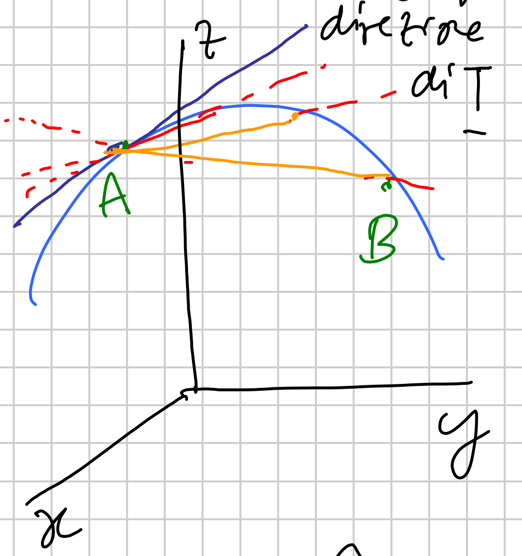
$$= (\Delta\lambda)^2 \left[\left(\frac{df_1}{d\lambda}\right)^2 + \left(\frac{df_2}{d\lambda}\right)^2 + \left(\frac{df_3}{d\lambda}\right)^2\right]$$

$$\overline{AB} \simeq \sqrt{\left(\frac{df_1}{d\lambda}\right)^2 + \left(\frac{df_2}{d\lambda}\right)^2 + \left(\frac{df_3}{d\lambda}\right)^2} \cdot \Delta\lambda$$



$$L = \int_{\lambda_1}^{\lambda_2} \sqrt{\left(\frac{df_1}{d\lambda}\right)^2 + \left(\frac{df_2}{d\lambda}\right)^2 + \left(\frac{df_3}{d\lambda}\right)^2} \, d\lambda$$

$$\sqrt{\left(\frac{df_1}{d\lambda}\right)^2+\left(\frac{df_2}{d\lambda}\right)^2+\left(\frac{df_3}{d\lambda}\right)^2}\cdot d\lambda = ds =$$ "lunghezza di un arco di curva infinitesimo"



$\lambda_B = \lambda_A + \Delta\lambda$

Come si determina la direzione di $\underline{T}$

$$\underline{T} = T_x\hat{\imath} + T_y\hat{\jmath} + T_z\hat{k}$$

$$\lim_{\Delta\lambda\to 0}\frac{f_1(\lambda_A+\Delta\lambda)-f_1(\lambda_A)}{\Delta\lambda} = \left.\frac{df_1}{d\lambda}\right|_{\lambda_A}$$

lo stesso succede per le altre direzioni

$$\underline{T} = \left.\frac{df_1}{d\lambda}\right|_{\lambda_A}\hat{\imath} + \left.\frac{df_2}{d\lambda}\right|_{\lambda_A}\hat{\jmath} + \left.\frac{df_3}{d\lambda}\right|_{\lambda_A}\hat{k}$$

tangente alla curva nel punto A

$$OA = f_1(\lambda_A)\hat{\imath} + f_2(\lambda_A)\hat{\jmath} + f_3(\lambda_A)\hat{k}$$

$$\hat{t} = \frac{\underline{T}}{|\underline{T}|} = \frac{\left.\frac{df_1}{d\lambda}\right|_{\lambda_A}\hat{\imath} + \left.\frac{df_2}{d\lambda}\right|_{\lambda_A}\hat{\jmath} + \left.\frac{df_3}{d\lambda}\right|_{\lambda_A}\hat{k}}{\sqrt{\left(\frac{df_1}{d\lambda}\right)^2_{\lambda=\lambda_A} + \left(\frac{df_2}{d\lambda}\right)^2_{\lambda=\lambda_A} + \left(\frac{df_3}{d\lambda}\right)^2_{\lambda=\lambda_A}}}$$

VERSORE TANGENTE

$$\underline{V} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$
$$|\underline{V}|^2 = \underline{V} \cdot \underline{V} = v_x^2 + v_y^2 + v_z^2$$
$$|\underline{V}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$v(t) \leftrightarrow v(s(t))$

È possibile cambiare la parametrizzazione di una curva $OP(\lambda)$ $OP(\lambda(\mu))$

$\uparrow$ parametro λ $\quad$ $\uparrow$ parametro μ

$R > 0$

$$\begin{cases} x = R\cos(t) \\ y = R\sin(t) \end{cases} \qquad \begin{cases} x = R \cdot \cos\left(\frac{u}{R}\right) \\ y = R \cdot \sin\left(\frac{u}{R}\right) \end{cases}$$

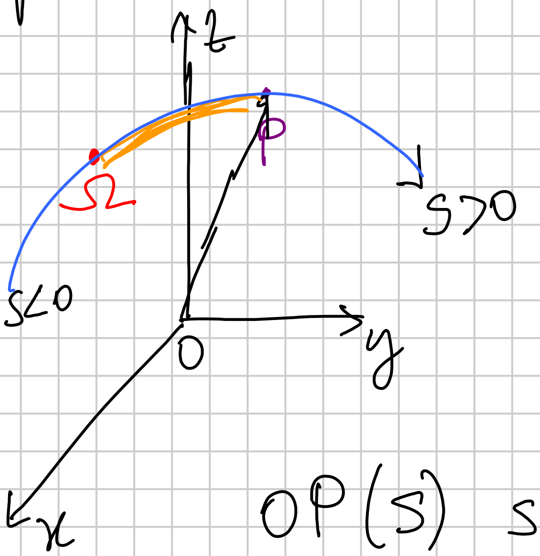
$\downarrow$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = $$
$$= \sqrt{R^2 \operatorname{sen}^2 t + R^2 \cos^2 t}\, dt = R\, dt$$

$$ds = \sqrt{\cancel{R^2} \cdot \operatorname{sen}^2\left(\frac{u}{R}\right) \cdot \frac{1}{\cancel{R^2}} + \cdots \cancel{R^2} \cos^2\left(\frac{u}{R}\right) \cdot \frac{1}{\cancel{R^2}}}\, du = du$$

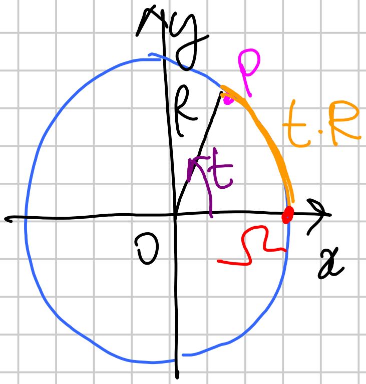
$$ds = du$$

PARAMETRIZZAZIONE NATURALE (s)



$OP(s)$ se s è proprio la lunghezza (a meno del segno) di $\overset{\frown}{\Omega P}$

Il parametro naturale è detto solitamente s e si chiama ASCISSA CURVILINEA



$$S = t \cdot R$$

$$t = \frac{S}{R}$$

$$\begin{cases} x = R\cos(t) \\ y = R\sin(t) \end{cases}$$

$$OP = R\left(\cos\left(\frac{S}{R}\right)\hat{\imath} + \operatorname{sen}\left(\frac{S}{R}\right)\hat{\jmath}\right)$$

$$\hat{t} = \frac{R\left(-\frac{1}{R}\sin\left(\frac{S}{R}\right)\hat{\imath} + \frac{1}{R}\cos\left(\frac{S}{R}\right)\hat{\jmath}\right)}{\underbrace{R\sqrt{\frac{1}{R^2}\sin^2\left(\frac{S}{R}\right) + \frac{1}{R^2}\cos^2\left(\frac{S}{R}\right)}}_{=1}}$$

$$\hat{t} = \frac{dOP}{dS} \qquad T = \frac{dOP}{d\lambda} \qquad OP(\lambda(S))$$

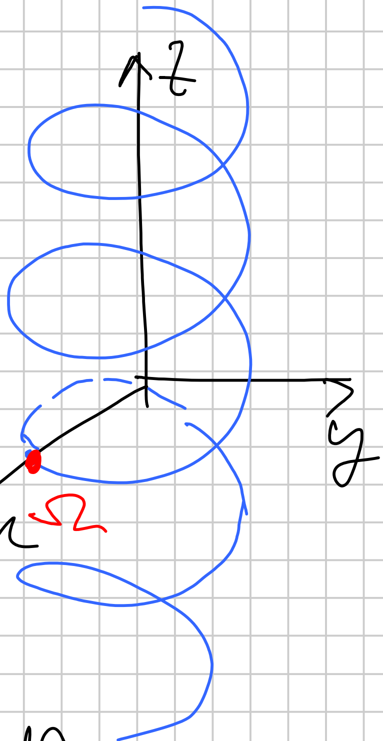
$$\hat{t} = \frac{dOP}{d\lambda} \cdot \frac{d\lambda}{dS}$$

ELICA CILINDRICA

$$\begin{cases} x = R\cos(\lambda) \\ y = R\sin(\lambda) \\ z = \frac{p\lambda}{2\pi} \end{cases}$$

$R, p > 0$ fissati

$\lambda \in \mathbb{R}$ parametro



$$ds = \sqrt{R^2\sin^2(\lambda) + R^2\cos^2(\lambda) + \frac{p^2}{4\pi^2}}\, d\lambda$$

$$= \underbrace{\sqrt{R^2 + \frac{p^2}{4\pi^2}}}_{R'}\, d\lambda \qquad ds = R'\, d\lambda$$

$OP(0) = R\hat{\imath}$

$$\int_0^s d\sigma = R_c \int_0^\lambda d\ell \qquad s = R_c \lambda \qquad \lambda = \frac{s}{R_c}$$

$$\begin{cases} x = R\cos\left(\frac{s}{R_c}\right) \\ y = R\sin\left(\frac{s}{R_c}\right) \\ z = \frac{p \cdot s}{2\pi R'} \end{cases}$$

$$\hat{t}(s) = \frac{dOP}{ds} = -\frac{R}{R'}\sin\Big(\underbrace{\frac{s}{R'}}_{\lambda}\Big)\hat{\imath} + \frac{R}{R'}\cos\left(\frac{s}{R'}\right)\hat{\jmath} + \frac{p}{2\pi R'}\hat{k}$$

$$|\hat{t}| = \sqrt{\frac{R^2}{R'^2} + \frac{p^2}{4\pi^2 R'^2}} = \sqrt{\frac{R^2 \cdot 4\pi^2}{4\pi^2R^2 + p^2} + \frac{p^2 \cdot \cancel{4\pi^2}}{\cancel{4\pi^2}(4\pi^2R^2+p^2)}} =$$

$$= \sqrt{\frac{4\pi^2R^2+p^2}{4\pi^2R^2+p^2}} = 1$$

$$\hat{t}(\lambda) = -\frac{R}{R'}\sin(\lambda)\hat{\imath} + \frac{R}{R'}\cos(\lambda)\hat{\jmath} + \frac{p}{2\pi R'}\hat{k}$$

1) $ds = \sqrt{}\, d\lambda$

2) si fissa un'origine Ω e si ricava $s = f(\lambda)$

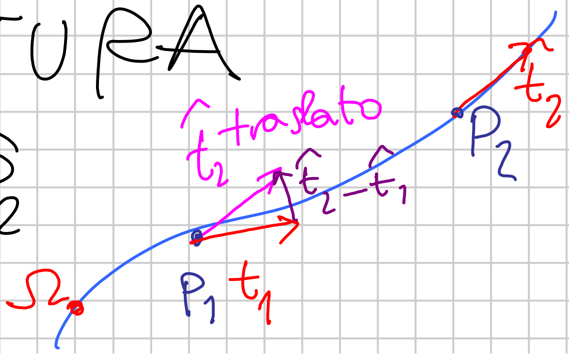
3) si inverte la relazione di sopra $\lambda = f^{-1}(s)$ che si sostituisce nella parametrizzazione iniziale

$s = \lambda + \frac{1}{2}\sin\lambda \qquad \lambda = f^{-1}(s)$

CURVATURA

Arco di curva $\widehat{P_1P_2}$

l'angolo fra $\hat{t}_1$ e $\hat{t}_2$



è detto angolo di contingenza ε relativo all'arco $\widehat{P_1P_2}$

$S_1 =$ lunghezza di $\widehat{\Omega P_1}$ $\quad S_2 =$ lunghezza di $\widehat{\Omega P_2}$

$\Delta S = S_2 - S_1$

$\underline{\Delta t} = \hat{t}_2 - \hat{t}_1 = \hat{t}(S_2) - \hat{t}(S_1) = \hat{t}(S_1 + \Delta S) - \hat{t}(S_1)$

$$\lim_{\Delta S \to 0} \left| \frac{\Delta t}{\Delta S} \right| = \left| \frac{d\hat{t}}{dS} \right|_{S = S_1}$$

$$\lim_{\Delta S \to 0} \left| \frac{\hat{t}(S_1 + \Delta S) - \hat{t}(S_1)}{\Delta S} \right| = \lim_{\Delta S \to 0} \frac{1}{|\Delta S|} \cdot \left| \hat{t}(S_1 + \Delta S) - \hat{t}(S_1) \right|$$

Come si esprime questa quantità in funzione di ε

$|\hat{t}(S_1 + \Delta S) - \hat{t}(S_1)| = 2 \operatorname{sen} \frac{\varepsilon}{2}$

$\Delta S \simeq \varepsilon$ quando $\Delta S \to 0$ anche $\varepsilon \to 0$ con lo stesso ordine

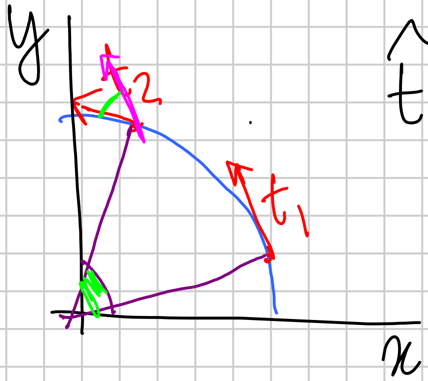
infinitesimo dello stesso ordine

$$\lim_{\Delta S \to 0} \frac{2 \operatorname{sen} \frac{\varepsilon}{2}}{|\Delta S|} = \lim_{\Delta S \to 0} \frac{2 \operatorname{sen} \varepsilon/2}{\varepsilon} \cdot \frac{\varepsilon}{|\Delta S|} = \lim_{\Delta S \to 0} \frac{\varepsilon}{|\Delta S|} = C$$

$\to 1$

$C =$ curvatura di γ nel punto P_1 corrispondente a S_1

$\hat{t} = -\operatorname{sen}\left(\frac{S}{R}\right) \hat{\imath} + \cos\left(\frac{S}{R}\right) \hat{\jmath}$



$$\hat{t}_2 - \hat{t}_1 = \left[-\sin\left(\frac{s_2}{R}\right) + \sin\left(\frac{s_1}{R}\right)\right]\hat{\imath} + \left[\cos\left(\frac{s_2}{R}\right) - \cos\left(\frac{s_1}{R}\right)\right]\hat{\jmath}$$

$$|\underline{\Delta t}|^2 = \left[1 + 1 - 2\,\mathrm{sen}\left(\frac{s_2}{R}\right)\mathrm{sen}\left(\frac{s_1}{R}\right) - 2\cos\left(\frac{s_2}{R}\right)\cos\left(\frac{s_1}{R}\right)\right]$$

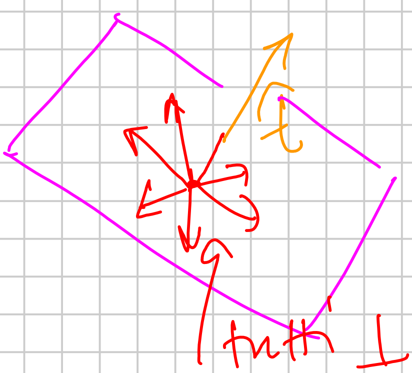
$$= 2 + 2\cos\left(\frac{s_2 - s_1}{R}\right) = 2 + 2\cos\left(\frac{\Delta s}{R}\right)$$

$$\lim_{\Delta s \to 0} \frac{\sqrt{2 + 2\cos\frac{\Delta s}{R}}}{\Delta s} = \frac{1}{R} \qquad \varepsilon = \frac{\Delta s}{R}$$

$$\lim_{\Delta s \to 0} \frac{\varepsilon}{R\varepsilon} = \frac{1}{R} \qquad \Delta s = R\varepsilon$$

$$c = \frac{1}{R} \qquad \frac{1}{c} = \text{raggio di curvatura} = R_c = \rho$$

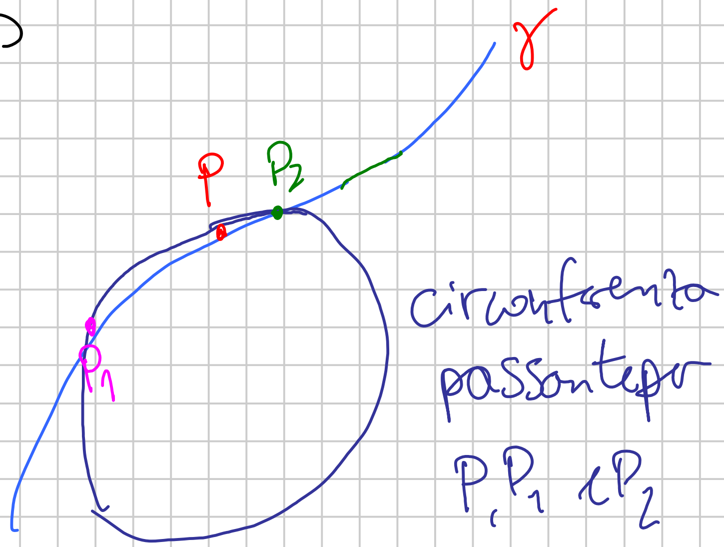
$$\lim_{\Delta s \to 0} \frac{\hat{t}(s + \Delta s) - \hat{t}(s)}{\Delta s} = \underline{N} \quad \text{vettore normale principale}$$



$\underline{N}$ è la direzione di una perpendicolare ben precisa

$\underline{N}$ è sempre diretto verso la concavità della curva

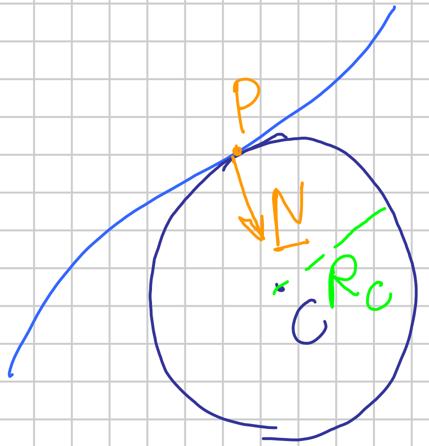
Il piano su cui stanno $\hat{t}_P$ ed $\hat{N}_P$ è detto PIANO OSCULATORE della curva γ in un punto P



$P_1 \to P$

$P_2 \to P$

Circonferenza che si ottiene effettuando il passaggio al limite è detta circonferenza osculatrice



Se γ è una curva piana, il piano osculatore coincide col piano su cui giace γ

Se il piano osculatore non varia al variare di P, allora la curva è piana

$$\underline{N} = \frac{d\hat{t}}{ds}$$

adimensionale

lunghezza

lunghezza^{-1}

$$|\underline{N}| = c \qquad \left|\frac{d\hat{t}}{ds}\right| = c = \frac{1}{R_c} \qquad |\underline{N}| \neq 1$$

$$\hat{n} = \frac{\underline{N}}{|\underline{N}|} = \frac{d\hat{t}/ds}{|d\hat{t}/ds|} = R_C \frac{d\hat{t}}{ds}$$

$$\boxed{\frac{d\hat{t}}{ds} = \frac{\hat{n}}{R_C}}$$

1ª formula di FRENET SERRET

$$\hat{t}(\lambda) \qquad \frac{d\hat{t}}{ds} = \frac{d\hat{t}}{d\lambda} \cdot \frac{d\lambda}{ds}$$

$$\frac{d f(\lambda(s))}{ds} = \frac{df}{d\lambda} \frac{d\lambda}{ds} \qquad ds = R_C d\lambda$$

$$\frac{df}{ds} = \frac{df}{d\lambda} \cdot \frac{1}{R_C}$$ per l'elica

$$\hat{n} = \frac{d\hat{t}}{ds} \cdot R_C \qquad |\hat{n}| = R_C \cdot \left|\frac{d\hat{t}}{ds}\right|$$

$$1$$

$$R_C = \frac{1}{\left|\frac{dt}{ds}\right|} = \frac{1}{\sqrt{\left(\frac{dt_x}{ds}\right)^2 + \left(\frac{dt_y}{ds}\right)^2 + \left(\frac{dt_z}{ds}\right)^2}} =$$

$$= \frac{1}{\sqrt{\left(\frac{d^2x}{ds}\right)^2 + \left(\frac{d^2y}{ds}\right)^2 + \left(\frac{d^2z}{ds}\right)^2}}$$

ATTENZIONE: bisogna derivare rispetto a S!

$\forall$ punto P di una curva "liscia" abbiamo 2 versori $\hat{t}, \hat{n}$ $\qquad \hat{t} \perp \hat{n} \qquad \hat{n} = \frac{d\hat{t}}{ds} R_c \qquad |t| = 1$

Completiamo la coppia per ottenere una terna $\hat{b}, \hat{t}, \hat{n}$ è destrorsa

$$\hat{b} = \hat{t} \wedge \hat{n}$$

$\hat{t}_P \quad \hat{n}_P \quad \hat{b}_P$ $\quad \forall P \in \gamma$ abbiamo una terna che è detta TERNA INTRINSECA

$\hat{b}$ è detto BINORMALE è $\perp$ al piano osculatore, ed è costante se la curva è piana

$$OP = R\cos\left(\frac{s}{\sqrt{R^2+\frac{p^2}{4\pi^2}}}\right)\hat{\imath} + R\sin\left(\frac{s}{\sqrt{R^2+\frac{p^2}{4\pi^2}}}\right)\hat{\jmath} + \frac{p\,s\,\hat{k}}{2\pi\sqrt{R^2+\frac{p^2}{4\pi^2}}}$$

$$\hat{t} = \frac{dOP}{ds} = \frac{R}{\sqrt{}}\left[-\sin\left(\frac{s}{\sqrt{}}\right)\hat{\imath} + \cos\left(\frac{s}{\sqrt{}}\right)\hat{\jmath}\right] + \frac{p}{2\pi\sqrt{}}\hat{k}$$

$$\underline{N} = \frac{d\hat{t}}{ds} = \frac{R}{R^2+\frac{p^2}{4\pi^2}}\left[-\cos\left(\frac{s}{\sqrt{}}\right) - \mathrm{sen}\left(\frac{s}{\sqrt{}}\right)\hat{\jmath}\right] + 0\hat{k}$$

$$|\underline{N}| = \frac{R}{R^2+\frac{p^2}{4\pi^2}} \qquad \hat{n} = \frac{\underline{N}}{|\underline{N}|} = -\cos\left(\frac{s}{\sqrt{}}\right)\hat{\imath} - \sin\left(\frac{s}{\sqrt{}}\right)\hat{\jmath}$$

$$R_c^{-1} = \left|\frac{d\hat{t}}{ds}\right| = \frac{R}{R^2+\frac{p^2}{4\pi^2}} \qquad R' = \sqrt{R^2+\frac{p^2}{4\pi^2}}$$

$$R_c = \frac{R'^2}{R}$$

$$\hat{b} = \hat{t} \wedge \hat{n} = \left\{\frac{R}{\sqrt{}}\left[-\mathrm{sen}\left(\frac{s}{\sqrt{}}\right)\hat{\imath} + \cos\left(\frac{s}{\sqrt{}}\right)\hat{\jmath}\right] + \frac{p}{2\pi\sqrt{}}\hat{k}\right\} \wedge$$

$$\left(-\cos\frac{s}{r}\hat{i} - \text{sen}\frac{s}{r}\hat{j}\right)$$

$$\frac{R}{r}\underbrace{\text{sen}^2\left(\frac{s}{r}\right)\hat{k} + \frac{R}{r}\cos^2}_{=1}\left(\frac{s}{r}\right)\hat{k} - \frac{P}{2\pi r}\cos\left(\frac{s}{r}\right)\hat{j}$$

$$+\frac{P}{2\pi r}\text{sen}\left(\frac{s}{r}\right)\hat{i}$$

$$\hat{b} = +\frac{P}{2\pi r}\left(\text{sen}\left(\frac{s}{r}\right)\hat{i} - \cos\left(\frac{s}{r}\right)\hat{j}\right) + \frac{R}{r}\hat{k}$$

Per ogni curva regolare è possibile definire una terna intrinseca - i versori $\hat{t}, \hat{n}, \hat{b}$ cambiano al variare del punto P sulla curva, ma mantengono per ogni punto le loro proprietà caratteristiche

- $\hat{t}_p$ tangente alla curva in P
- $\hat{n}_p$ normale principale alla curva in P

} Piano osculatore, circonferenza osculatrice, Raggio di curvatura

- $\hat{b}_p$ indica se la curva è piana oppure no

$$\begin{cases} x = R\cos\left(\frac{s}{R}\right) \\ y = R\sin\left(\frac{s}{R}\right) \end{cases}$$

$$\hat{t} = -\sin\left(\frac{s}{R}\right)\hat{i} + \cos\left(\frac{s}{R}\right)\hat{j}$$

$$\hat{n} = -\cos\left(\frac{s}{r}\right)\hat{i} - \text{sen}\left(\frac{s}{R}\right)\hat{j}$$

$$\hat{b} = \hat{k}$$