MOTI CENTRALI

Titolo nota 15/10/2012

$OP \parallel \underline{a}_P \qquad OP \wedge \underline{v}_P = \underline{h}$

$\downarrow$

$OP \wedge \underline{a}_P = \underline{0}$

$$\frac{d}{dt}(OP \wedge \underline{v}_P) = \cancel{\underline{v}_P \wedge \underline{v}_P} + OP \wedge \underline{a}_P = \underline{0}$$

$$\frac{d}{dt}(OP \wedge \underline{v}_P) = 0 \rightarrow OP \wedge \underline{v}_P = \underline{h}$$

$\underline{h} = c\,\hat{h}$ $\hat{h}$ è un versore $\perp$ al piano su cui avviene il moto di P

(C'è la possibilità che il moto sia rettilineo (r) e $O \in$ alla retta r $\Rightarrow$ $\nexists$ un unico piano su cui avviene il moto $\quad \underline{h} = 0$, $\hat{h}$ non è definito, $c = 0$)

$$\dot{A} = \frac{1}{2}\underbrace{(OP \wedge \underline{v})}_{c\hat{h}} \cdot \hat{h} = \frac{1}{2}c\,\underbrace{\hat{h} \cdot \hat{h}}_{1} = \frac{1}{2}c$$

$\shortparallel$

$$\dot{A} = \frac{1}{2}\rho^2\dot{\theta} \qquad \frac{1}{2}\rho^2\dot{\theta} = \frac{1}{2}c = \text{costante}$$

II LEGGE DI KEPLERO

In un moto centrale, la velocità areolare è costante

$$\underline{a} = (\ddot\rho - \rho\dot\theta^2)\hat r + \frac{1}{\rho}\boxed{\frac{d}{dt}(\underbrace{\rho^2\dot\theta}_{2\dot A = c}) = 0}\,\hat\theta$$

in un moto centrale

$$a_\theta = \frac{1}{\rho}\frac{d}{dt}(c) = 0$$

→ in un moto centrale non c'è accelerazione trasversale a_θ, c'è solo a_ρ

$$m\rho^2\dot\theta = L_0 = \text{momento angolare} \qquad mc = L_0$$

$$(\underline{L}\, k_0)$$

$$c = \rho^2\dot\theta = [\rho(0)]^2\cdot\dot\theta(0)$$

note le condizioni iniziali, è noto anche c

$$\boxed{\dot\theta = \frac{c}{\rho^2}}$$

c'è un legame fra $\dot\theta$ e ρ

$$\rho(\theta(t))$$

$$v_\rho = \dot\rho = \frac{d\rho}{dt} = \frac{d\rho}{d\theta}\cdot\frac{d\theta}{dt} = \frac{d\rho}{d\theta}\cdot\dot\theta = \frac{d\rho}{d\theta}\cdot\frac{c}{\rho^2}$$

$$\frac{d}{d\theta}\left(\frac{1}{\rho}\right) = -\frac{1}{\rho^2}\frac{d\rho}{d\theta} = -\frac{v_\rho}{c}$$

$$f(x)$$

$$\frac{d}{dx}\left(\frac{1}{f(x)}\right) = -\frac{f'(x)}{[f(x)]^2}$$

$v_\theta = \rho\dot\theta = \dfrac{c}{\rho}$	$v_\rho = -c\,\dfrac{d(1/\rho)}{d\theta}$

$$a_\rho = \ddot{\rho} - \rho\dot{\theta}^2 = \frac{d}{dt}(\dot{\rho}) - \rho\dot{\theta}^2 = \frac{d\dot{\rho}}{d\theta}\cdot\frac{d\theta}{dt} - \rho\dot{\theta}^2 =$$

$$= \frac{d\dot{\rho}}{d\theta}\cdot\dot{\theta} - \cancel{\rho}\frac{c^2}{\rho^{\cancel{4}\,3}} = \dot{\theta}\frac{d}{d\theta}\left(-c\frac{d(1/\rho)}{d\theta}\right) - \frac{c^2}{\rho^3} = \qquad \dot{\theta} = \frac{c}{\rho^2}$$

$$= -c\dot{\theta}\frac{d^2(1/\rho)}{d\theta^2} - \frac{c^2}{\rho^3} = -\frac{c^2}{\rho^2}\frac{d^2}{d\theta^2}(1/\rho) - \frac{c^2}{\rho^3} =$$

$$\frac{c}{\rho^2}$$

$$\boxed{= -\frac{c^2}{\rho^2}\left[\frac{1}{\rho} + \frac{d^2}{d\theta^2}(1/\rho)\right] = a_\rho}$$

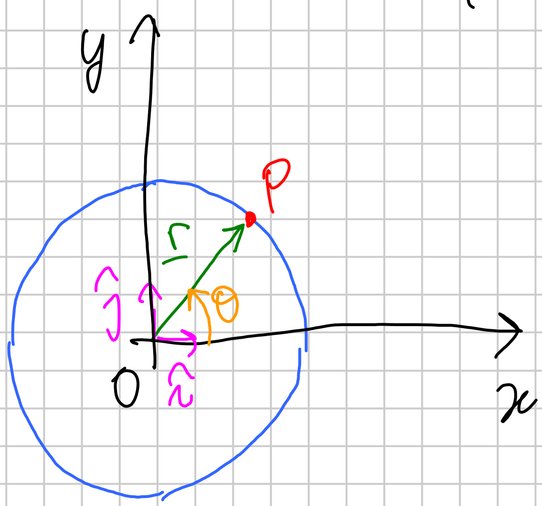
FORMULA DI BINET

$a_\rho(\theta)$ dipende solo dalla conoscenza di $\rho(\theta)$

Non c'è bisogno di conoscere $\rho(t)$ per poter calcolare a_ρ, basta $\rho(\theta)$

NOTO a_ρ si ricava $\rho(\theta)$ (traiettoria)

MOTO CIRCOLARE



$|OP| = |\underline{r}| = \text{costante} = R$

Per descrivere il moto di P basta conoscere $\theta(t)$

$$OP(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath} =$$

$$= R\cos(\theta(t))\,\hat{\imath} + R\sin(\theta(t))\,\hat{\jmath}$$

$\dot{\theta} = \frac{d\theta}{dt}$ = velocità angolare scalare

$s = R\theta \qquad \dot{s} = R\dot{\theta} \qquad \ddot{s} = R\ddot{\theta}$

$$\underline{v}_P = \frac{d\underline{r}}{dt} = -R\dot{\theta}\sin(\theta(t))\,\hat{\imath} + R\dot{\theta}\cos(\theta(t))\,\hat{\jmath} =$$

$$= R\dot{\theta}\underbrace{\left(-\sin(\theta)\hat{\imath} + \cos(\theta)\hat{\jmath}\right)}_{\hat{t}} = R\dot{\theta}\hat{t} \qquad \rho = R$$

$v_\rho = 0 \qquad v_\theta = \rho\dot{\theta} \qquad v_\rho = \dot{\rho} \qquad \rho = R \rightarrow \dot{\rho} = 0 \qquad \underline{v}_P = R\dot{\theta}\hat{\theta}$

IN QUESTO CASO, MA NON SEMPRE, $\boxed{\begin{array}{l}\hat{\theta} = \hat{t} \\ -\hat{r} = \hat{n}\end{array}}$

$$\underline{a} = \frac{d}{dt}(R\dot{\theta}\hat{\theta}) = \overset{\ddot{s}}{R\ddot{\theta}}\,\hat{\theta} - \overset{\dot{s}^2/R}{R\dot{\theta}^2}\,\hat{r}$$

$$= \ddot{s}\hat{\theta} - \frac{\dot{s}^2}{R}\hat{r} = \ddot{s}\hat{t} + \frac{\dot{s}^2}{R}\hat{n}$$

$$|\underline{a}| = \sqrt{R^2\ddot{\theta}^2 + R^2\dot{\theta}^4} = R\sqrt{\ddot{\theta}^2 + \dot{\theta}^4}$$

$a_\rho = -R\dot{\theta}^2 \qquad a_\theta = R\ddot{\theta}$

Se $\ddot{\theta} = 0$, abbiamo $a_\theta = 0$ $\quad a_\rho =$ costante

$\theta(t) = \dot{\theta}_0 \cdot t + \theta_0$ MOTO CIRCOLARE UNIFORME

$$\begin{cases} x(t) = R\cos(\dot{\theta}_0 t + \theta_0) \\ y(t) = R\sin(\dot{\theta}_0 t + \theta_0) \end{cases} \qquad \begin{cases} \dot{x} = -\dot{\theta}_0 y \\ \dot{y} = +\dot{\theta}_0 x \end{cases} \qquad \begin{cases} \ddot{x} = -\dot{\theta}_0^2 x \\ \ddot{y} = -\dot{\theta}_0^2 y \end{cases}$$

$\underline{r} = x\hat{\imath} + y\hat{\jmath} \qquad \ddot{\underline{r}} = -\dot{\theta}_0^2\,\underline{r}$ $\quad$ MCU è un moto periodico e centrale

$\exists\, T : \underline{r}(t+T) = \underline{r}(t) \quad \forall t \in \mathbb{R}$

il minimo $T \neq 0$ che realizza questa condizione (se esiste!) è detto PERIODO

$$T = \frac{2\pi}{|\dot{\theta}_0|}$$
$\uparrow$
(s)

$$\frac{1}{T} = \nu = \text{frequenza (Hz)}$$
$$= \frac{\omega}{2\pi} \leftarrow \text{frequenza angolare o pulsazione} \left(\frac{rad}{s}\right)$$

$\dot{\theta}_0 t + \theta_0$ = fase all'istante t (radianti)

θ_0 = fase iniziale fase

MOTO ARMONICO

Deriva dal MCU $OP = x(t)\hat{\imath} + y(t)\hat{\jmath}$

Consideriamo la proiezione P' di P sull'asse delle x $OP' = x(t)\hat{\imath}$

$\omega = \dot{\theta}_0$

$$x(t) = R(\cos(\omega t + \theta_0))$$
$$\dot{x}(t) = -\omega R \cdot \sin(\omega t + \theta_0)$$
$$\ddot{x}(t) = -\omega^2 R \cdot \cos(\omega t + \theta_0)$$

$$\ddot{x} = -\omega^2 x$$

R = ampiezza del moto armonico

O = centro del moto armonico

$$\boxed{\ddot{x} + \omega^2 x = 0} \; *$$

Ogni moto armonico semplice, non smorzato, non forzato è descritto da un'equazione di questo tipo

$*$ $x = A\cos(\omega t + \alpha)$

$$\begin{cases} \ddot{x} + \omega^2 x = 0 \\ x(0) = x_0 \\ \dot{x}(0) = \dot{x}_0 \end{cases}$$

$$x = C_1 \cos(\omega t) + C_2 \sin(\omega t)$$

$$\dot{x} = -\omega C_1 \sin(\omega t) + \omega C_2 \cos(\omega t)$$

$$\begin{cases} x_0 = C_1 + 0 \\ \dot{x}_0 = 0 + \omega C_2 \end{cases} \rightarrow \begin{cases} C_1 = x_0 \\ C_2 = \dfrac{\dot{x}_0}{\omega} \end{cases}$$

$$x(t) = x_0 \cos(\omega t) + \frac{\dot{x}_0}{\omega} \sin(\omega t)$$

$$A = \sqrt{x_0^2 + \frac{\dot{x}_0^2}{\omega^2}} \qquad \cos\alpha = \frac{x_0}{A} \qquad \text{sen}\,\alpha = \frac{-\dot{x}_0}{\omega A}$$

$$y = \boxed{a \cos x + b \sin x} = \sqrt{a^2+b^2}\left(\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\cos\alpha} \cos x + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{-\sin\alpha} \sin x\right)$$

$$= \sqrt{a^2+b^2}\,(\cos x \cos\alpha - \text{sen}\,x\, \text{sen}\,\alpha)$$

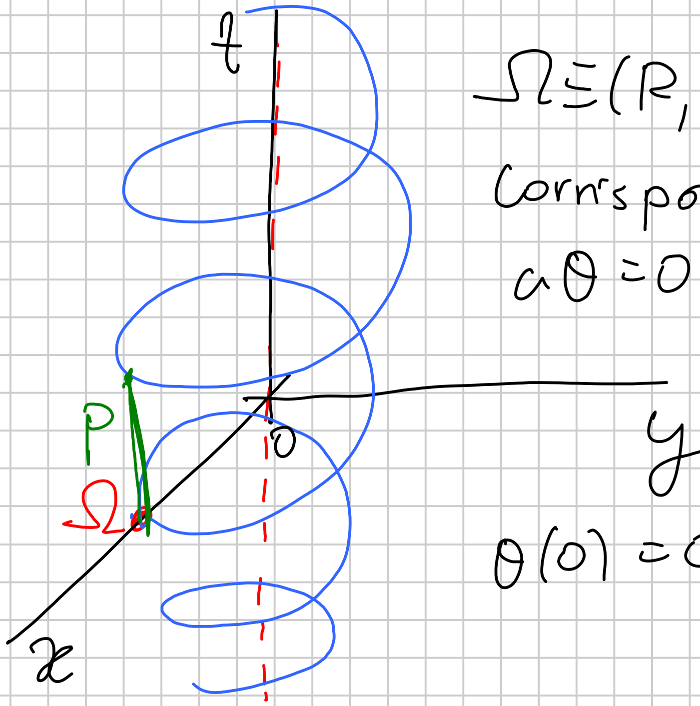
$$= \sqrt{a^2+b^2} \cos(x+\alpha)$$

1) Il M.A. approssima i moti vibratori quando A è piccolo

2) Con l'analisi di Fourier è possibile rappresentare OGNI fenomeno periodico con somme (infinite) di moti armonici, ognuno dei quali ha frequenze, ampiezze e fasi opportunamente definite

MOTO SU UN'ELICA

$$\begin{cases} x = R\cos(\theta(t)) \\ y = R\sin(\theta(t)) \\ z = \dfrac{p\cdot\theta(t)}{2\pi} \end{cases}$$



$\Omega \equiv (R,0,0)$

Corrispondente a $\theta = 0$

$\theta(0) = 0$

$$\begin{cases} \dot{x} = -R\dot{\theta}\sin(\theta(t)) \\ \dot{y} = +R\dot{\theta}\cos(\theta(t)) \\ \dot{z} = \dfrac{p\dot{\theta}}{2\pi} \end{cases}$$

$$\begin{cases} \ddot{x} = -R\ddot{\theta}\sin\theta - R\dot{\theta}^2\cos(\theta) \\ \ddot{y} = +R\ddot{\theta}\cos(\theta) - R\dot{\theta}^2\sin(\theta) \\ \ddot{z} = \dfrac{p\ddot{\theta}}{2\pi} \end{cases}$$

$$ds = \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 + \left(\frac{dz}{d\theta}\right)^2}\, d\theta = \overbrace{\boxed{\sqrt{R^2 + \frac{p^2}{4\pi}}}}^{R'}\, d\theta = R'\, d\theta$$

$s(t) = R'\theta(t) + C$ $\qquad \dot{s}(t) = R'\dot{\theta}(t)$ $\qquad \ddot{s} = R'\ddot{\theta}(t)$

$\underline{v} = \dot{s}\hat{t} = R'\dot{\theta}(t)\hat{t}$

$$R_c^{-1} = \sqrt{\left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 + \left(\frac{d^2z}{ds^2}\right)^2} \cdots\cdots \frac{R}{R^2 + \frac{p^2}{4\pi^2}}$$

$$R_c = \frac{R^2 + p^2/4\pi^2}{R} = R + \frac{p^2}{4\pi^2 R}$$

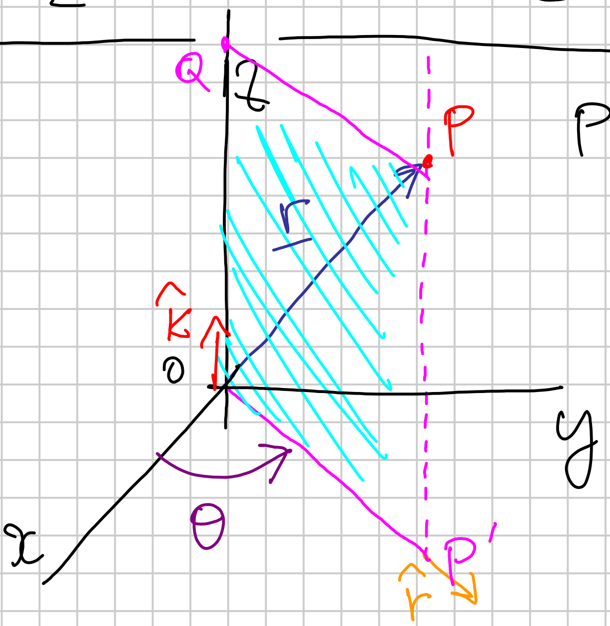
$$\underline{a} = \ddot{s}\hat{t} + \frac{\dot{s}^2}{R_c}\hat{n} = R'\ddot{\theta}\hat{t} + \frac{R'^2\dot{\theta}^2}{R_c}\hat{n} =$$

$$= R'\ddot{\theta}\hat{t} + \cancel{\left(R^2 + \frac{p^2}{4\pi^2}\right)}\frac{R\dot{\theta}^2}{\cancel{\left(R^2 + \frac{p^2}{4\pi^2}\right)}}\hat{n} = R'\ddot{\theta}\hat{t} + R\dot{\theta}^2\hat{n}$$

$\theta(t) = \omega t \qquad \dot{\theta} = \omega \qquad \ddot{\theta} = 0$ moto uniforme

$\underline{v} = R'\omega\hat{t} \qquad \underline{a} = R\omega^2\hat{n}$

COORDINATE CILINDRICHE E SFERICHE



$P \equiv (x, y, z)$

Rettangolo $OP'PQ$

$$\underline{r} = OP' + z\hat{k} = \rho\hat{r} + z\hat{k}$$

$$\hat{r} = \cos\theta\,\hat{\imath} + \operatorname{sen}\theta\,\hat{\jmath}$$

$$\rho = \sqrt{x^2 + y^2} = |OP'|$$

$$\frac{OP'}{\rho} = \hat{r}$$

$$\underline{r} = \rho\hat{r} + z\hat{k}$$

$\hat{r}$ non è fisso

$\hat{k}$ è fisso

$$\underline{v}_P = \frac{d\underline{r}}{dt} = \dot{\rho}\hat{r} + \rho\dot{\theta}\hat{\theta} + \dot{z}\hat{k}$$

$$\underline{a}_P = \frac{d\underline{v}_P}{dt} = (\ddot{\rho} - \rho\dot{\theta}^2)\hat{r} + (2\dot{\theta}\dot{\rho} + \ddot{\theta}\rho)\hat{\theta} + \ddot{z}\hat{k}$$

Se $\rho =$ costante $\quad \sqrt{x^2+y^2} =$ costante $\quad x^2+y^2 =$ costante

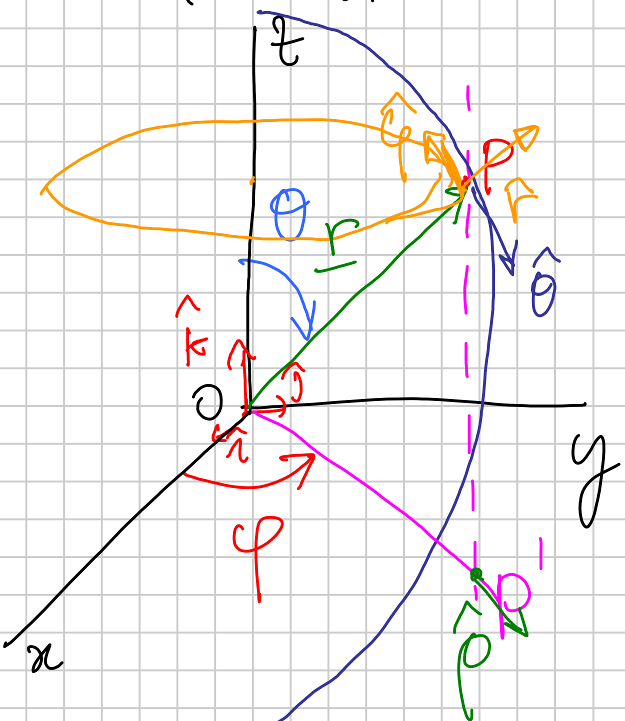
il moto di P avviene su un cilindro d'eq. $x^2+y^2=\rho^2$

Se $z =$ costante il moto è piano, il piano è parallelo al piano xy

Se $\theta =$ costante il moto di P è piano e avviene nel piano di eq. $-\text{sen}\theta\, x + \cos\theta\, y = 0$

il vettore $-\text{sen}\theta\, \hat{\imath} + \cos\theta\, \hat{\jmath}$ è $\perp$ a OP $\forall t$

COORDINATE SFERICHE



θ è l'angolo fra $\hat{k}$ e $\underline{r}$

φ è l'angolo fra $\hat{\imath}$ e OP'

$P \equiv (x, y, z)$

$r = |\underline{r}|$

$z = r\cos\theta \qquad |OP'| = r\,\text{sen}\theta$

$$\begin{cases} x = r\,\text{sen}\theta\cos\varphi \\ y = r\,\text{sen}\theta\,\text{sen}\varphi \\ z = r\,\text{sen}\theta \end{cases} \qquad \begin{matrix} 0 \le \theta \le \pi \\ 0 \le \varphi < 2\pi \end{matrix}$$

definiamo $\hat{r} = \dfrac{\underline{r}}{r}$

$$\underline{r} = r\left(\text{sen}\theta\cos\varphi\,\hat{\imath} + \text{sen}\theta\,\text{sen}\varphi\,\hat{\jmath} + \text{sen}\theta\,\hat{k}\right) = r\hat{r}$$

$$\underline{v}_P = \frac{dr}{dt} = \frac{d}{dt}(r\hat{r}) = \dot{r}\hat{r} + r\frac{d\hat{r}}{dt}$$

$$\hat{\varphi} = -\operatorname{sen}\varphi\,\hat{\imath} + \cos\varphi\,\hat{\jmath} \qquad \hat{\varphi}\cdot\hat{k} = 0$$

$$\hat{\varphi}\cdot\hat{r} = -\operatorname{sen}\varphi(\operatorname{sen}\theta\cos\varphi) + \cos\varphi(\operatorname{sen}\theta\operatorname{sen}\varphi) + 0\cdot\operatorname{sen}\theta$$
$$= 0$$

$\hat{\theta} = ?$ $\hat{r}, \hat{\theta}, \hat{\varphi}$ devono formare una terna destrorsa

$$\hat{\theta} = \hat{\varphi}\wedge\hat{r} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -\operatorname{sen}\varphi & \cos\varphi & 0 \\ \operatorname{sen}\theta\cos\varphi & \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\theta \end{vmatrix} =$$

$$(\cos\theta\cos\varphi)\,\hat{\imath} + (\cos\theta\operatorname{sen}\varphi)\,\hat{\jmath} - \operatorname{sen}\theta\,\hat{k}$$

$$\frac{d\hat{r}}{dt} = \frac{d}{dt}\left(\operatorname{sen}\theta\cos\varphi\,\hat{\imath} + \operatorname{sen}\theta\operatorname{sen}\varphi\,\hat{\jmath} + \cos\theta\,\hat{k}\right) =$$

$$= \left(\dot{\theta}\cos\theta\cos\varphi - \dot{\varphi}\operatorname{sen}\theta\operatorname{sen}\varphi\right)\hat{\imath}$$
$$+ \left(\dot{\theta}\cos\theta\operatorname{sen}\varphi + \dot{\varphi}\operatorname{sen}\theta\cos\varphi\right)\hat{\jmath} - \dot{\theta}\operatorname{sen}\theta\,\hat{k}$$

$$= +\dot{\varphi}\operatorname{sen}\theta\overbrace{\left(-\operatorname{sen}\varphi\,\hat{\imath} + \cos\varphi\,\hat{\jmath}\right)}^{\hat{\varphi}} + \dot{\theta}\underbrace{\left(\cos\theta\cos\varphi\,\hat{\imath} + \cos\theta\operatorname{sen}\varphi\,\hat{\jmath} - \operatorname{sen}\theta\,\hat{k}\right)}_{\hat{\theta}} = \dot{\varphi}\operatorname{sen}\theta\,\hat{\varphi} + \dot{\theta}\,\hat{\theta}$$

$$\underline{v}_P = \dot{r}\hat{r} + r\dot{\varphi}\,\mathrm{sen}\,\theta\,\hat{\varphi} + r\dot{\theta}\hat{\theta}$$

$$\underline{a}_P = \frac{d\underline{v}_P}{dt}$$

$$\frac{d\hat{\varphi}}{dt} = \frac{d}{dt}\left(-\mathrm{sen}\,\varphi\,\hat{\imath} + \cos\varphi\,\hat{\jmath}\right) = -\dot{\varphi}\left(\cos\varphi\,\hat{\imath} + \mathrm{sen}\,\varphi\,\hat{\jmath}\right)$$

$$= -\dot{\varphi}\,\hat{\rho} \qquad \left(\hat{\rho} = \frac{OP'}{|OP'|}\right)$$

$$\frac{d\hat{\theta}}{dt} = \frac{d}{dt}\left(\cos\theta\cos\varphi\,\hat{\imath} + \cos\theta\,\mathrm{sen}\,\varphi\,\hat{\jmath} - \mathrm{sen}\,\theta\,\hat{k}\right) =$$

$$= \left(-\dot{\theta}\,\mathrm{sen}\,\theta\cos\varphi - \dot{\varphi}\cos\theta\,\mathrm{sen}\,\varphi\right)\hat{\imath}$$

$$+ \left(-\dot{\theta}\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi + \dot{\varphi}\cos\theta\cos\varphi\right)\hat{\jmath}$$

$$- \dot{\theta}\cos\theta\,\hat{k}$$

$$= -\dot{\theta}\,\overbrace{\left(\mathrm{sen}\,\theta\cos\varphi\,\hat{\imath} + \mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,\hat{\jmath} + \cos\theta\,\hat{k}\right)}^{\hat{r}}$$

$$+ \dot{\varphi}\cos\theta\,\big(\underbrace{-\mathrm{sen}\,\varphi\,\hat{\imath} + \cos\varphi)\,\hat{\jmath}}_{\hat{\varphi}}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r} + \dot{\varphi}\cos\theta\,\hat{\varphi}$$

$$\underline{a}_P = \frac{dv_P}{dt} = \frac{d}{dt}\left(\dot{r}\hat{r} + r\dot{\varphi}\,\mathrm{sen}\,\theta\,\hat{\varphi} + r\dot{\theta}\hat{\theta}\right) =$$

$$\ddot{r}\hat{r} + \dot{r}\frac{d\hat{r}}{dt} + \dot{r}\dot{\varphi}\,\mathrm{sen}\,\theta\,\hat{\varphi} + r\ddot{\varphi}\,\mathrm{sen}\,\theta\,\hat{\varphi} + r\dot{\varphi}\dot{\theta}\cos\theta\,\hat{\varphi} +$$

$$+ r\dot\varphi \,\text{sen}\theta \frac{d\hat\varphi}{dt} + \dot r \dot\theta \hat\theta + r\ddot\theta\hat\theta + r\dot\theta \frac{d\hat\theta}{dt}$$

$$\ddot r \hat r + \dot r\left(\dot\theta\hat\theta + \text{sen}\theta\,\dot\varphi\hat\varphi\right) + \left(\dot r\dot\varphi\,\text{sen}\theta + r\ddot\varphi\,\text{sen}\theta + r\dot\varphi\dot\theta\cos\theta\right)\hat\varphi$$

$$- r\dot\varphi^2 \text{sen}\theta\,\hat\rho + (\dot r\dot\theta + r\ddot\theta)\hat\theta + r\dot\theta\left(-\dot\theta\hat r + \dot\varphi\cos\theta\hat\varphi\right)$$

$$(\ddot r - r\dot\theta^2)\hat r - r\dot\varphi^2\text{sen}\theta\,\hat\rho + (2\dot r\dot\theta + r\ddot\theta)\hat\theta$$

$$+ \left[2\dot r\dot\varphi\,\text{sen}\theta + r\ddot\varphi\,\text{sen}\theta + 2r\dot\varphi\dot\theta\cos\theta\right]\hat\varphi$$

$\hat\rho$ espresso in funzione di $\hat r$ e $\hat\theta$

$$\hat\rho = \cos\varphi\,\hat\imath + \text{sen}\varphi\,\hat\jmath$$

$$\hat r = \text{sen}\theta\cos\varphi\,\hat\imath + \text{sen}\theta\,\text{sen}\varphi\,\hat\jmath + \cos\theta\,\hat k$$

$$\hat\theta = \cos\theta\cos\varphi\,\hat\imath + \cos\theta\,\text{sen}\varphi\,\hat\jmath - \text{sen}\theta\,\hat k$$

$$\boxed{\text{sen}\theta\,\hat r + \cos\theta\,\hat\theta} = \cos\varphi\,\hat\imath + \text{sen}\varphi\,\hat\jmath = \hat\rho$$

$$\underline{a_P} = \left(\ddot r - r\dot\theta^2 - r\dot\varphi^2\text{sen}^2\theta\right)\hat r + \left(r\ddot\theta + 2\dot r\dot\theta - r\dot\varphi^2\text{sen}\theta\cos\theta\right)\hat\theta + \left[(r\ddot\varphi + 2\dot r\dot\varphi)\text{sen}\theta + 2r\cos\theta\,\dot\theta\dot\varphi\right]\hat\varphi$$

se r = costante moto su una sfera

se φ = costante e r = costante moto su un meridiano

se θ = costante e r = costante moto su un parallelo